Culminating Review for Calculus

- Limits
- Derivatives
- Velocity and Acceleration
- Extreme Values
- Optimization
- Curve Sketching

0011

Limits

$$\lim_{x \to a} f(x) = L$$

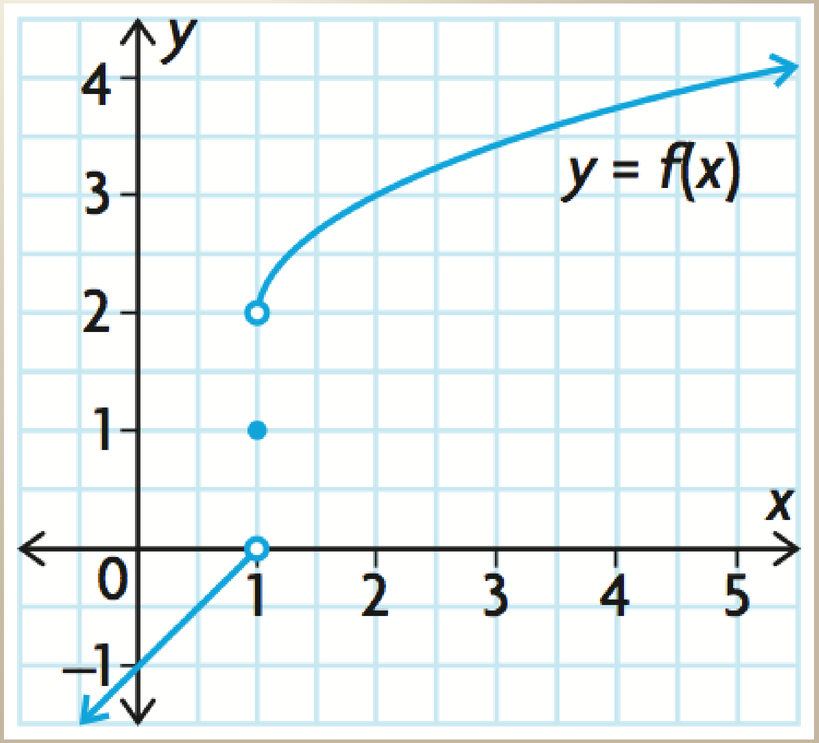
Tells us that the value of $f(x)$ approaches L as x approaches the value a.

(a) $\lim_{x \to 1^-} f(x) =$

(b) $\lim_{x \to 1^+} f(x) =$

(c) $\lim_{x \to 1} f(x) =$

(d) $f(1) =$



- Algebraic methods

$$\lim_{x\to 0}(3x^2 - 2x + 7)$$

direct substitution

$$\lim_{x\to -4}\left(\frac{x^2 + 7x + 12}{x + 4}\right)$$

factor and simplify

$$\lim_{x\to 0}\left(\frac{\sqrt{x+9} - 3}{x}\right)$$

multiply by the conjugate $\sqrt{x+9} + 3$

or use substitution, let $u = \sqrt{x+9}$

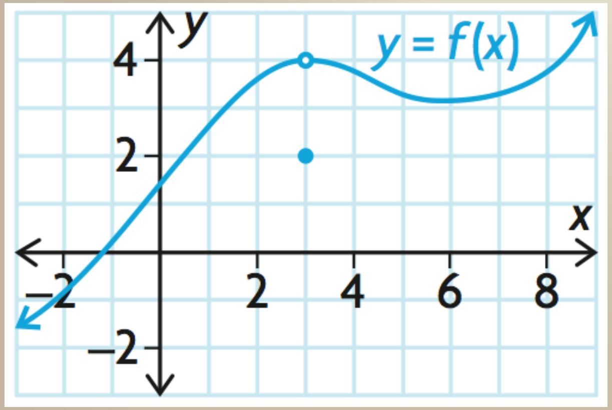
$$\lim_{x\to 0}\left(\frac{\sqrt[3]{x+27} - 3}{x}\right)$$

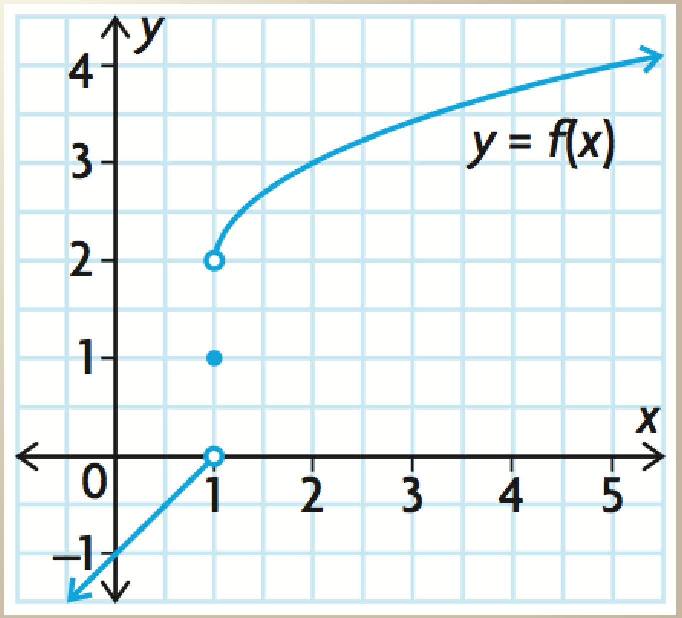
use substitution, let $u = \sqrt[3]{x+27}$

Continuity

- A function f is continuous at $x = a$ if

1. $\lim_{x \to a} f(x)$ exists
2. $\lim_{x \to a} f(x) = f(a)$





Derivatives

Derivative of a function using *first principles*:

$$f'(x) = \lim_{h \to 0} \left(\frac{f(x+h) - f(x)}{h} \right)$$

Power Rule

$$f(x) = x^n \qquad f'(x) = nx^{n-1}$$

Power of a Function Rule

$$f(x) = \left(g(x)\right)^n \qquad f'(x) = n\left(g(x)\right)^{n-1} g'(x)$$

Product Rule

$$f(x) = p(x)q(x) \qquad f'(x) = p'(x)q(x) + p(x)q'(x)$$

Quotient Rule

$$f(x) = \frac{p(x)}{q(x)} \qquad f'(x) = \frac{p'(x)q(x) - p(x)q'(x)}{\left(q(x)\right)^2}$$

Chain Rule

$$f(x) = g\left(h(x)\right) \qquad f'(x) = g'\left(h(x)\right)h'(x)$$

Exponential Rules

$$f(x) = e^x \qquad f'(x) = e^x$$

$$f(x) = e^{g(x)} \qquad f'(x) = e^{g(x)} g'(x)$$

$$f(x) = b^x \qquad f'(x) = b^x \ln b$$

$$f(x) = b^{g(x)} \qquad f'(x) = b^{g(x)} (\ln b)\, g'(x)$$

Trigonometric Rules

$$f(x) = \sin x \qquad f'(x) = \cos x$$

$$f(x) = \cos x \qquad f'(x) = -\sin x$$

$$f(x) = \sin\big(g(x)\big) \qquad f'(x) = \cos\big(g(x)\big)\, g'(x)$$

$$f(x) = \cos\big(g(x)\big) \qquad f'(x) = -\sin\big(g(x)\big)\; g'(x)$$

Velocity and Acceleration

<u>Position, velocity and acceleration</u>

- Given position as $s(t)$ then
  - Velocity is $v(t) = s'(t)$
  - Acceleration is $a(t) = v'(t) = s''(t)$

Extreme Values

Algorithm for Finding Maximum and Minimum (Extreme) Values:

If a function $f(x)$ has a derivative at every point in the interval $a \leq x \leq b$, calculate $f(x)$ at:

- all points in the interval $a \leq x \leq b$, where $f'(x) = 0$ (critical values)
- the endpoint $x = a$ and $x = b$

The maximum value of $f(x)$ on the interval $a \leq x \leq b$, is the largest of these values, and the minimum value of $f(x)$ on the interval is the smallest of these values

Optimization

An Algorithm for Solving Optimization Problems

- Identify the variables.
- Determine a function that represents the quantity to be optimized.
- Determine the domain of the function to be optimized.
- Use the algorithm for extreme values to find the absolute maximum or minimum in the domain.
- Conclude.

An Algorithm for Curve Sketching

Using $f(x)$

- Determine any intercepts.
- Determine any *vertical asymptotes* and determine the behaviour on either side of the asymptote.
- Determine any *horizontal or oblique asymptotes*. Identify if the function values approach the asymptote from above or below.

Using $f'(x)$

- Determine any critical numbers by finding where $f'(x) = 0$ or where $f'(x)$ is undefined.
- Determine the intervals of increase/decrease, and identify any local or absolute extrema.

Using $f''(x)$

- Determine possible points of inflection by finding where $f''(x) = 0$ or where $f''(x)$ is undefined.
- Determine the intervals of concavity, and identify any points of inflection.
- Sketch the curve.