MCV4U – Exam Review Day 1

Multiple Choice:

1. Given the points B(-3, 1, -2) and C(3, -5, 10) the vector $\overline{BC}$ in component form is
   A. [6, -6, 12] B. [-6, 6, -12] C. [-5, -4, 8] D. [1, -1, 2] E. [-1, 1, -2]
   A

2. Given $|\vec{a}| = 5$, $|\vec{b}| = 6$ and the angle between them is $60°$, then $\vec{a} \bullet \vec{b} =$
   A. 15 B. 30 C. 51.96 D. 5.5 E. none of these
   A

3. If $\vec{d} \times \vec{h} = [2,-6,7]$, then $\vec{h} \times \vec{d} =$
   A. $[2,-6,7]$ B. $[7,-6,2]$ C. $[-2,6,-7]$ D. $[-6,7,2]$ E. none of these
   C

4. Which of the following are parametric equations of a line through (3,-2) and (9,-4)?
   A. $x = 3 + 9t$, $y = -2 - 4t$ B. $x = 3 - 9t$, $y = -2 + 4t$ C. $x = -3 + 6t$, $y = +2 - 2t$ D. $x = 9 + 3t$, $y = -4 - t$ E. none of these
   D

5. Which of the following points is on the plane $3x + y - 4z - 5 = 0$?
   A. (3, 1 -4) B. (2, -1, 0) C. (1, -3, -1) D. (-3, 7, 2) E. none of these
   B

6. Which of the following yields a vector perpendicular to both original vectors?
   A. dot product B. scalar product C. cross product E. none of these
   C

7. Which of the following is a vector quantity?
   A. time B. force C. mass D. temperature E. none of these
   B

8. Using the ***Triangle Law of Addition*** to add two vectors, the vectors must be drawn
   A. tail to head B. head to head C. tail to tail
   D. head to tail E. backwards
   D

Short Answer:

1. Given the vectors $\vec{p} = [3,-1,4]$ and $\vec{u} = [-2,5,1]$. Find the angle between the vectors $\vec{p}$ and $\vec{u}$.

$$\vec{p} \bullet \vec{u} = 3(-2) - 1(5) + 4(1) = -7$$

$$|\vec{p}| = \sqrt{3^2 + (-1)^2 + 4^2} = \sqrt{26}$$

$$|\vec{u}| = \sqrt{(-2)^2 + 5^2 + 1^2} = \sqrt{30}$$

$$\cos\theta = \frac{\vec{p} \bullet \vec{u}}{|\vec{p}||\vec{u}|}$$

$$\cos\theta = \frac{-7}{\sqrt{26}\sqrt{30}}$$

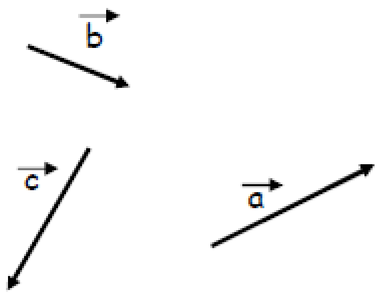
$$\cos\theta = \frac{-7}{\sqrt{780}}$$

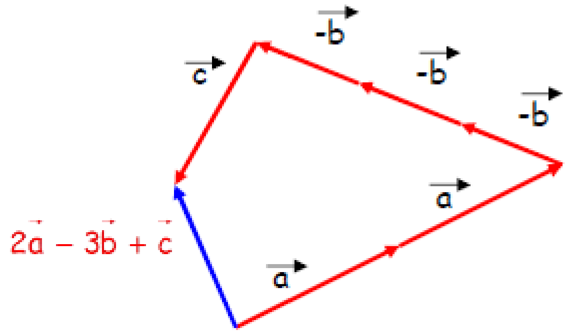
$$\cos\theta \doteq -0.2506$$

$$\therefore \theta \doteq 104.5°$$

The angle between the vectors is about 104.5°.

2. Use a diagram to draw $2\vec{a} - 3\vec{b} + \vec{c}$ (to scale) given the vectors shown.





3. Prove $\vec{a} \bullet \vec{b} = \vec{b} \bullet \vec{a}$ algebraically in 3 - space.

Let $\bar{a} = [a_1, a_2, a_3]$ and $\bar{b} = [b_1, b_2, b_3]$

$\bar{a} \bullet \bar{b} = [a_1, a_2, a_3] \bullet [b_1, b_2, b_3]$ and $\bar{b} \bullet \bar{a} = [b_1, b_2, b_3] \bullet [a_1, a_2, a_3]$

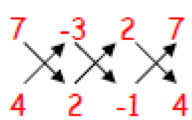
$= a_1b_1 + a_2b_2 + a_3b_3$ $\qquad$ $= b_1a_1 + b_2a_2 + b_3a_3$

$\qquad = a_1b_1 + a_2b_2 + a_3b_3$

$\therefore \ \bar{a} \bullet \bar{b} = \bar{b} \bullet \bar{a}$ in $\mathbb{R}^3$

4. Find a vector perpendicular to $\bar{b} = [2, 7, -3]$ and $\bar{c} = [-1, 4, 2]$.

$$\bar{b} \times \bar{c} = [14 + 12, 3 - 4, 8 + 7]$$
$$= [26, -1, 15]$$

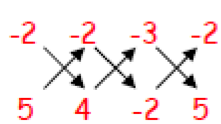


5. Find vector and Cartesian equations of the plane through the points (11, -2, 3), (8, -4, 1), and (6, 1, 5).

$$\bar{d}_1 = [8 - 11, -4 + 2, 1 - 3] = [-3, -2, -2]$$
$$\bar{d}_2 = [6 - 8, 1 + 4, 5 - 1] = [-2, 5, 4]$$

Vector Equation: $[x, y, z] = [11, -2, 3] + t[-3, -2, -2] + s[-2, 5, 4]$

$$\bar{d}_1 \times \bar{d}_2 = [-8 + 10, 4 + 12, -15 - 4]$$
$$= [2, 16, -19]$$



Cartesian Equation: $\quad 2x + 16y - 19z + D = 0$

$$2(11) + 16(-2) - 19(3) + D = 0$$
$$22 - 32 - 57 + D = 0$$
$$\therefore D = 67$$

A Cartesian equation is 2x + 16y - 19z + 67 = 0

6. Find parametric equations for the line through the points (7, 0, 5) and (-2, 4, 3).

$\overline{d} = [7 - -2, 0 - 4, 5 - 3] = [9, -4, 2]$ is direction vector for this line.

Parametric Equations:

$x = 7 + 9t$, $y = -4t$, $z = 5 + 2t$ or $x = -2 + 9t$, $y = 4 - 4t$, $z = 3 + 2t$

7. Describe all the ways a line and a plane may exist in 3-Space and the types of intersections they may or may not have.

A line may intersect a plane in a unique point.

A line may lie on the plane. All points on the line are also on the plane. The line is said to be coincident with the plane.

The line may be parallel and distinct from the plane, sharing no common points with the plane.

8. Find the intersections of the following 3 planes. If the solution is infinite, also find one particular solution. Make sure you describe the type of solution. All work **MUST** be shown. You may use a calculator to check your work, but your methodology has to be demonstrated in your solution.

$$x + 3y + 3z - 8 = 0$$
$$x - y + 3z - 4 = 0$$
$$2x + 6y + 6z - 16 = 0$$

$$x + 3y + 3z = 8$$
$$x - y + 3z = 4$$

Subtract $4y = 4$

$\therefore y = 1$

$2 \times (x + 3y + 3z = 16)$ ---->$2x + 6y + 6z = 16$

$2x + 6y + 6z = 16$ ---------->$2x + 6y + 6z = 16$

Subtracting $0 = 0$

This implies an infinite number of solutions.

$x + 3y + 3z - 8 = 0$

Let $z = t$

$x + 3 + 3t = 8$

$\therefore x = 5 - 3t$

These planes intersect in the line $x = 5 - 3t$, $y = 1$, $z = t$.

A particular solution (let t = 1) is $x = 5 - 3(1) = 2$, $y = 1$, $z = 1$. Therefore (2,1,1) is on all 3 planes.