MCV4U – Exam Review Day 3

Multiple Choice:

1. The derivative of the function $y = f(x)$ where $x = 3$ is D

A. $\lim_{\Delta x \to 0} \frac{f(x+a) - f(x)}{\Delta x}$
B. $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$
C. $\lim_{h \to 0} \frac{f(x+3) - f(x)}{h}$
D. $\lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$
E. none of these

2. The Power Differentiation Rule is A

A $\frac{d}{dx}(x^n) = nx^{n-1}$
B. $\frac{d}{dx}(k) = 0,\ k \in R$
C. $\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$, if u is a function of x
D. $\frac{d}{dx}(u^n) = nu^{n-1}$
E. none of these

3. If $f(x) = r(x)h(x)$ then $f'(x) =$ D

A. $r(x)h'(x) - h(x)r'(x)$
B. $r'(x)h'(x)$
C. $h(x)r'(x) - r(x)h'(x)$
D. $r'(x)h(x) + h'(x)r(x)$
E. none of these

4. If $y = f(g(x))$, then $\frac{dy}{dx} =$ E

A. $f(g'(x))$
B. $f(g'(x))g(x)$
C. $f'(g(x))$
D. $f'(g'(x))$
E. none of these

5. The graph of the function $y = f(x)$ is always concave up where D

A. $f'(x) = 0$
B. $f''(x) = 0$
C. $f''(x) < 0$
D. $f''(x) > 0$
E. $f'(x) \le 0$

6. $\frac{d}{dx}\left[2e^{3x}\right] =$ D

A. $2e^{3x}$
B. e^{3x}
C. $2e^{x}$
D. $6e^{3x}$
E. none of these

7. If $y = \sin 5x$, then $y' =$ D

A. $-\cos 5x$
B. $5\cos x$
C. $-5\cos 5x$
D. $5\cos 5x$
E. none of these

8. What kind of line shows an instantaneous rate of change the best? B

A. a secant line
B. a tangent line

9. $\frac{d}{dx}\left[5^{4x^2}\right] =$ B

A. $8x5^{4x^2}$
B. $8x(\ln 5)5^{4x^2}$
C. $4x^2 5^{4x^2-1}$
D. $8x5^{4x^2-1}$
E. $(\ln 5)5^{8x}$

10. An interval upon which a function has a negative derivative it is said to be _______. D

A. concave up
B. increasing
C. concave down
D. decreasing
E. none of these

11. An interval upon which a function has a positive second derivative the function is said to be __________. | A |

A. concave up B. increasing C. concave down
D. decreasing E. none of these

12. Which of the following is **not** a proper symbol for the derivative? | C |

A. y' B. $\frac{dy}{dx}$ C. $\frac{d}{dx}$ D. D_x E. $h'(x)$

Short Answer:

1. What is an average rate of change? Give an example.

An average rate of change is a rate of change over a horizontal distance or period of time that is between two points. It is defined as the amount of vertical change (dependent variable) divided by the horizontal change (independent variable). For example, the average speed of an object between 2 and 5 seconds. Here the period of time is 3 seconds so this would be called an average rate of change.

2. The weasel population (in hundreds) in an area is modeled for the next 8 years by the function $P(t) = -2t^2 + 16t + 5$.

a) What is the instantaneous rate of change of the population at 2 years?
b) What is happening to the population at 2 years? What do you think might be causing this change in the population?

a) $P'(t) = \lim_{h\to 0}\frac{P(t+h)-P(t)}{h}$

$P'(t) = \lim_{h\to 0}\frac{-2(t+h)^2+16(t+h)+5-(-2t^2+16t+5)}{h}$

$P'(t) = \lim_{h\to 0}\frac{-2t^2-4th-2h^2+16t+16h+5+2t^2-16t-5}{h}$

$P'(t) = \lim_{h\to 0}\frac{-4th-2h^2+16h}{h}$

$P'(t) = \lim_{h\to 0} -4t-2h+16$

$P'(t) = -4t+16$

$P'(2) = -4(2)+16$

$P'(2) = 8$ hundred weasels/year

b) At 2 years the population is increasing by 800 weasels per year. This might be caused by the local predator population has declined as part of a normal cycle.

3. Find the equation of the tangent line to the function $y = 4x^3 - 5x^2 + 8x$ where $x = 1$.

$y' = 12x^2 - 10x + 8$

$y'_{x=1} = 12(1)^2 - 10(1) + 8$

$y'_{x=1} = 10$

$y = mx + b$

$y = 10x + b$ [need to find y]

$y = 4(1)^3 - 5(1)^2 + 8(1)$

$y = 7$

$7 = 10(1) + b$

$\therefore b = -3$

The equation of the tangent is

$y = 10x - 3$

4. Find the first derivative for each of the following functions

a) $f(x) = 4x^7 - \sqrt{2}x^3 - 8x + 6$

$f'(x) = 28x^6 - 3\sqrt{2}x^2 - 8$

b) $f(x) = (2x^3 - 5x)^6$

$f'(x) = 6(2x^3 - 5x)^5(6x^2 - 5)$

$f'(x) = 6(6x^2 - 5)(2x^3 - 5x)^5$

c) $y = \dfrac{28x^3 + 14x^2 - 21x}{7x}$

$y = 4x^2 + 2x - 3$

$\dfrac{dy}{dx} = 8x + 2$

d) $f(x) = 7x^5\sqrt{3x^4 - 5x^2}$

$f(x) = 7x^5(3x^4 - 5x^2)^{\frac{1}{2}}$

$f'(x) = 35x^4(3x^4 - 5x^2)^{\frac{1}{2}} + 7x^5\frac{1}{2}(3x^4 - 5x^2)^{-\frac{1}{2}}(12x^3 - 10x)$

$f'(x) = 35x^4\sqrt{3x^4 - 5x^2} + \dfrac{42x^8 - 35x^6}{\sqrt{3x^4 - 5x^2}}$

$f'(x) = 35x^4\sqrt{3x^4 - 5x^2}\dfrac{\sqrt{3x^4 - 5x^2}}{\sqrt{3x^4 - 5x^2}} + \dfrac{42x^8 - 35x^6}{\sqrt{3x^4 - 5x^2}}$

$f'(x) = \dfrac{35x^4(3x^4 - 5x^2)}{\sqrt{3x^4 - 5x^2}} + \dfrac{42x^8 - 35x^6}{\sqrt{3x^4 - 5x^2}}$

$f'(x) = \dfrac{105x^8 - 175x^6}{\sqrt{3x^4 - 5x^2}} + \dfrac{42x^8 - 35x^6}{\sqrt{3x^4 - 5x^2}}$

$\therefore f'(x) = \dfrac{147x^8 - 210x^6}{\sqrt{3x^4 - 5x^2}}$

e) $f(x) = \dfrac{5}{(5x^2 + 8)^3}$

$f(x) = 5(5x^2 + 8)^{-3}$

$f'(x) = -15(5x^2 + 8)^{-4}(10x)$

$\therefore f'(x) = -\dfrac{150x}{(5x^2 + 8)^4}$

f) $f(x) = \sin(x^2 + 1)$

$f'(x) = \cos(x^2 + 1) \times 2x$

$f'(x) = 2x\cos(x^2 + 1)$

g) $y = 4e^{x^3}$

$y' = 4e^{x^3}\left(3x^2\right)$

$\therefore y' = 12x^2e^{x^3}$

h) $y = \left(e^{2x}\right)\sin\left(4x - 5\right)$

$\frac{dy}{dx} = \left(2e^{2x}\right)\sin\left(4x - 5\right) + e^{2x}4\cos\left(4x - 5\right)$

$\therefore \frac{dy}{dx} = \left(2e^{2x}\right)\sin\left(4x - 5\right) + 4e^{2x}\cos\left(4x - 5\right)$

i) $f\left(x\right) = 6^{4x}$

$f'\left(x\right) = 6^{4x} \times \ln\left(6\right) \times 4$

$\therefore f'\left(x\right) = 4\ln\left(6\right)6^{4x}$

j) $b\left(x\right) = 6\cos 4x^2$

$b'\left(x\right) = 6\left(-\sin 4x^2\right) \times 8x$

$\therefore b'\left(x\right) = -48x\sin 4x^2$