MCV4U
February 22, 2016

KNOWLEDGE	38	
APPLICATION	23	
COMMUNICATION		

Level

Name: Answers

Parent/Guardian Signature:

UNIT 1: RATES OF CHANGE & LIMITS

Communication reasoning, in writing and use of mathematical language, symbols and conventions will be assessed throughout this test.

KNOWLEDGE

1. Explain why the limit does not exist.
$\lim_{x \to 3} \frac{1}{x-3}$

From the left

x	$\frac{1}{x-3}$
2	−1
2.5	−2
2.9	−10
2.99	−100
2.999	−1000

From the right

x	$\frac{1}{x-3}$
4	1
3.5	2
3.1	10
3.01	100
3.001	1000

$\lim_{x \to 3^-} \frac{1}{x-3} = -\infty$ ✓

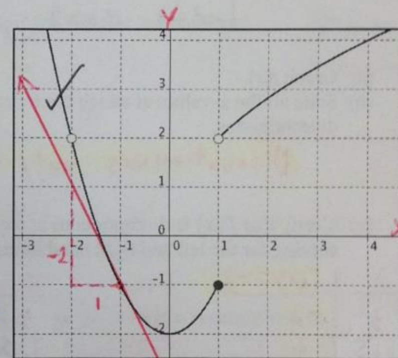
$\lim_{x \to 3^+} \frac{1}{x-3} = \infty$ ✓

∴ limit DNE

4 marks

2. Using the graph shown at the right, determine:
(a) $\lim_{x \to -2} g(x) =$ 2 ✓ (b) $\lim_{x \to 1^-} g(x) =$ −1 ✓
(c) $\lim_{x \to 1^+} g(x) =$ 2 ✓ (d) $\lim_{x \to 1} g(x) =$ DNE ✓
(e) $g(1) =$ −1 ✓
(f) Explain using 1 or more of the 3 algebraic conditions which must be met for a curve to be continuous.

$\boxed{\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) = f(a)}$ to be continuous ✓



8 marks

(g) Draw a tangent to the curve at x= -1 and determine its slope using the graph not it's equation.

slope $= \frac{\text{rise}}{\text{run}} = \frac{-2}{1}$ or -2 ✓

3. Evaluate each limit.
(a) $\lim_{x \to -1} \left(\frac{2x}{x-2}\right)$

$= \frac{2(-1)}{(-1)-2}$ ✓

$= \frac{-2}{-3}$

$= \frac{2}{3}$ ✓

2 marks

(b) $\lim_{x \to 2} \frac{2x^2 - x - 6}{3x^2 - 7x + 2}$

$\boxed{= \frac{2(2)^2 - (2) - 6}{3(2)^2 - 7(2) + 2} = \frac{0}{0} \text{ indeterminate}}$ ✓

$\lim_{x \to 2} \frac{(2x+3)\cancel{(x-2)}}{\cancel{(x-2)}(3x-1)}$ ✓

$\lim_{x \to 2} \frac{(2x+3)}{(3x-1)}$ ✓

$= \frac{2(2)+3}{3(2)-1}$ ✓

$= \frac{7}{5}$ or $1\frac{2}{5}$ ✓

5 marks

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(c) $\lim_{x\to2}\left(\frac{x^2-2x}{x-2}\right) = \frac{(2)^2-2(2)}{(2)-2}$ ✓

$= \frac{0}{0}$ indeterminate

$= \lim_{x\to2}\frac{x(\cancel{x-2})}{(\cancel{x-2})}$ ✓

$= \lim_{x\to2} x$ ✓

$= 2$ ✓

4 marks

(d) $\lim_{x\to5}\frac{x-5}{\sqrt{x-1}-2} = \frac{(5)-5}{\sqrt{5-1}-2} = \frac{0}{0}$ ✓ indeterminate

$= \lim_{x\to5}\frac{(x-5)}{\sqrt{x-1}-2}\cdot\frac{\sqrt{x-1}+2}{\sqrt{x-1}+2}$ ✓

$= \lim_{x\to5}\frac{(x-5)(\sqrt{x-1}+2)}{(x-1)-2^2}$ ✓

$= \lim_{x\to5}\frac{(\cancel{x-5})\cdot(\sqrt{x-1}+2)}{(\cancel{x-5})}$ ✓

$= \lim_{x\to5}\sqrt{x-1}+2$ ✓

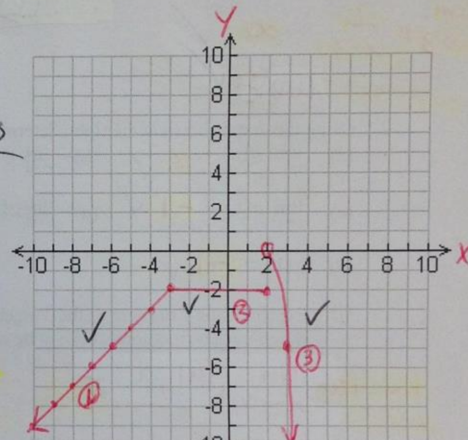
$= \sqrt{5-2}+2$

$= 4$ ✓

6 marks

4. Graph $f(x)=\begin{cases} x+1 & \text{if } x\le -3 \quad ① \\ -2 & \text{if } -3<x\le 2 \quad ② \\ -x^2+4 & \text{if } x>2 \quad ③ \end{cases}$

7marks



(a) Graph $f(x)$.
(b) State all the x-values at which the function is discontinuous.

Discontinuous only at $x=2$ ✓

(c) Verify that $f(x)$ is discontinuous at the x-values by solving for the left and right hand limits.

From the left

x	f(x) = -2
1	-2
1.5	-2
1.9	-2
1.99	-2
1.999	-2

✓

From the right

x	$f(x)=-x^2+4$
3	-5
2.5	-2.25
2.1	-0.41
2.01	-0.0401
2.001	-0.004001

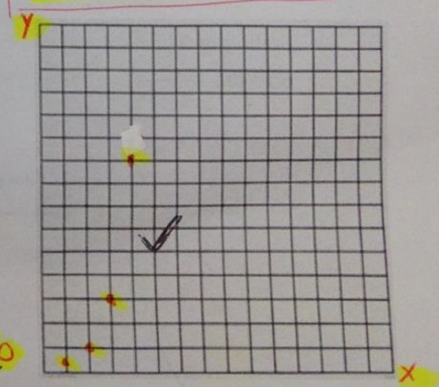
✓

$\therefore \lim_{x\to2^-} -2 \neq \lim_{x\to2^+} -x^2+4$ ✓

5. For each of the following sequences,
(a) State the limit, if it exists. If it does not exist, explain why. Use a graph to support your answer.
(b) Write a limit expression to represent the end behaviour of the sequence.

3 marks

i.) $\frac{1}{3}, 1, 3, 9, 27, \ldots, 3^{n-2}, \ldots$



a.) points: $(n, t_n) = (1, \frac{1}{3}), (2,1), (3,3), (4,9), (5,27)$ ...etc

The terms are not converging to a value, ✓ therefore the limit does not exist

b.) End behaviour is represented by $\lim_{n\to\infty} 3^{n-2} = \infty$

meaning the terms become larger positive values without bound so the limit does not exist ✓

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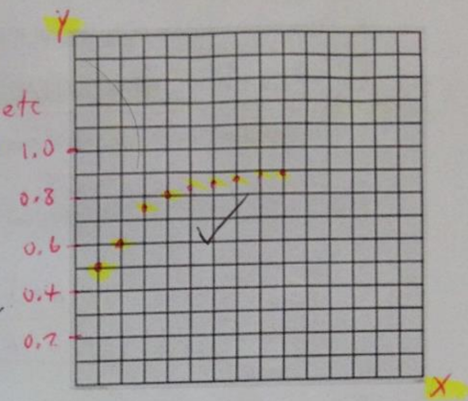
ii.) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \ldots, \frac{n}{n+1}, \ldots$

3 marks



a.) Points: $(n, t_n) = (1, 0.5), (2, 0.\bar{6}), (3, 0.75), (4, 0.80)$ ...etc
The terms seem to be converging to a value of 1, therefore the limit does exist. This can be verified by trying larger values of n eg/ (100, 0.99) & (1000, 0.999)

b.) End behaviour is represented by $\lim_{n \to \infty} \frac{n}{n+1} = 1$
meaning as the terms get larger, they approach the value of 1.

APPLICATION

6. A weather balloon is rising vertically. After t hours, its distance above the ground, measured in kilometres, is given by the formula $s(t) = 8t - t^2$

(a) Determine the average rate of change (*ie average velocity*) of the weather balloon from $t = 2$ h to $t = 5$ h.

9 marks

$s(2) = 8(2) - (2)^2$
$= 12$

$s(5) = 8(5) - (5)^2$
$= 15$

$\frac{\Delta s}{\Delta t} = \frac{15-12}{5-2}$
$= \frac{3}{3}$
$= 1$ Km/h

∴ the average ROC for 2s to 5s is 1 Km/h

(b) Determine the instantaneous rate of change (*ie instantaneous velocity*) in height at time $t = 3$ h.

From the left side

interval	$\Delta f(x)$	Δx	$\Delta f(x)/\Delta x$
$2 \le x \le 3$	3	1	3
$2.5 \le x \le 3$	1.25	0.5	2.5
$2.9 \le x \le 3$	0.21	0.1	2.1
$2.99 \le x \le 3$	0.0201	0.01	2.01

From the right side

interval	$\Delta f(x)$	Δx	$\Delta f(x)/\Delta x$
$3 \le x \le 4$	1	1	1
$3 \le x \le 3.5$	0.75	0.5	1.5
$3 \le x \le 3.1$	0.19	0.1	1.9
$3 \le x \le 3.01$	0.0199	0.01	1.99

From the left & right of t = 3, secants approach 2. The instantaneous ROC is the point (2, 12)

7. State whether each of the following can be represented by secant slope or tangent slope.

3 marks

a) Jordan grew 8 cm in 15 months
secant slope

b) When the radius of a circular ripple on the surface of a pond is 4 cm, the circumference of the ripple is increasing at 21.5 cm/s
tangent slope

c) The price of gas increased by 10cents/L at 3p.m.
tangent slope

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8. Determine the slope of the tangent at $x = 4$ for $f(x) = \frac{1}{x+1}$ using the first principles method.

7 marks

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{(x+h+1)} - \frac{1}{(x+1)}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{(x+1) - (x+h+1)}{(x+1)(x+h+1)}}{h}$$

$$= \lim_{h \to 0} \frac{-h}{(x+1)(x+h+1)} \times \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{-1}{(x+1)(x+h+1)}$$

$$= \lim_{h \to 0} \frac{-1}{(x+1)(x+(0)+1)}$$

$$= \frac{-1}{(x+1)^2}$$

$$\therefore f'(x) = \frac{-1}{(x+1)^2}$$

$$f'(4) = \frac{-1}{((4)+1)^2}$$

$$= \frac{-1}{25}$$

$\therefore$ the slope of the tangent at $x=4$ is $-\frac{1}{25}$

9. What is a possible function, f(x), if

a) $\frac{dy}{dx} = \lim_{h\to 0} \frac{-6(x+h)^3 - 2(x+h)^2 + 6x^3 + 2x^2}{h}$

possible
$f(x) = -6x^3 - 2x^2$

check $f(x+h) = -6(x+h)^3 - 2(x+h)^2$

$$\therefore \frac{f(x+h) - f(x)}{h}$$

$$= \frac{-6(x+h)^3 - 2(x+h)^2 - (-6x^3 - 2x^2)}{h}$$

$$= \frac{-6(x+h)^3 - 2(x+h)^2 + 6x^3 + 2x^2}{h}$$

$\therefore f(x) = -6x^3 - 2x^2$

4 marks

b) $f'(4) = \lim_{h\to 0} \frac{[5(4+h)^2 - 2] - [5(4)^2 - 2]}{h}$

possible
$f(x) = 5x^2 - 2$

check
$f(x+h) = 5(x+h)^2 - 2$

$$\therefore \frac{f(x+h) - f(x)}{h}$$

$$= \frac{5(x+h)^2 - 2 - (5x^2 - 2)}{h}$$

sub in $x = 4$

$$= \frac{[5(4+h)^2 - 2] - [(5(4)^2 - 2)]}{h}$$

$\therefore f(x) = 5x^2 - 2$

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