MCV4U
March 10, 2016

KNOWLEDGE		56
APPLICATION		34
COMMUNICATION		↓ Level

Name: ANSWERS

Parent/Guardian Signature: ____________________

UNIT 2: DERIVATIVE SHORT CUTS AND POSITION, VELOCITY & ACCELERATION

Communication reasoning, in writing and use of mathematical language, symbols and conventions will be assessed throughout this test.

KNOWLEDGE

25 marks

1. Differentiate each of the following. *Do not simplify your answer.*

a) $y = 7$

$\frac{dy}{dx} = 0$ ✓

b) $g(x) = 5x^4 - 3x^{-5}$

$g'(x) = 5(4)x^{4-1} - 3(-5)x^{-5-1}$ ✓ ✓

c) $f(x) = \frac{4x^{12}}{3}$

$f(x) = \frac{4}{3}(12)x^{12-1}$ ✓

d) $f(x) = (8x^3 - 4x)(3x^2 + x)$

$f'(x) = (8(3)x^{3-1} - 4(1)x^0)\cdot(3x^2+x) + (3(2)x^{2-1} + (1)x^{1-1})\cdot(8x^3-4x)$ ✓ ✓ ✓ ✓ ✓

e) $h(x) = (6x^2 - 4x)^5$

$h(x) = (5)\cdot(6x^2-4x)^{5-1}\cdot(6(2)x^{2-1} - 4(1)x^{1-1})$ ✓ ✓ ✓

f) $g(x) = \frac{8x^{\frac{1}{2}} - 6}{5x^{\frac{1}{5}} - 7}$

$g'(x) = \frac{(8(\frac{1}{2})x^{\frac{1}{2}-1} - 0)(5x^{\frac{1}{5}} - 7) - (5(\frac{1}{5})x^{\frac{1}{5}-1} - 0)\cdot(8x^{\frac{1}{2}} - 6)}{(5x^{\frac{1}{5}} - 7)^2}$ ✓ ✓ ✓

g) $m(x) = (5x-9)^3(6x^4 - 2x)^7$

$m'(x) = (3)\cdot(5x-7)^{3-1}(5(1)x^{1-1} - 0)\cdot(6x^4-2x)^7 + (7)(6x^4-2x)^{7-1}\cdot(6(4)x^{4-1} - 2(1)x^{1-1})\cdot(5x-9)^3$ ✓ ✓ ✓ ✓ ✓

h) $y = (5x+6)\sqrt{4 - x^2}$

$\frac{dy}{dx} = (5(1)x^{1-1} + 0)\cdot(4-x^2)^{\frac{1}{2}} + (\frac{1}{2})(4-x^2)^{\frac{1}{2}-1}\cdot(0 - (2)x^{2-1})\cdot(5x+6)$ ✓ ✓ ✓ ✓

2. Prove that the following is a derivative rule. Justify your answer with an example.

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

7 marks

Let $f(x) = 2x+1$ ✓
Let $g(x) = 3x+1$

$f(x) + g(x) = (2x+1) + (3x+1)$
$= 5x+2$ ✓

$\frac{d}{dx}(f(x)+g(x)) = 5x+2$
$= 5$ ✓

$\frac{d}{dx}f(x) = 2x+1$
$= 2$ ✓

$\frac{d}{dx}g(x) = 3x+1$
$= 3$ ✓

$\therefore \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$
$= 2+3$
$= 5$ ✓

✓ $\therefore \frac{d}{dx}[f(x)+g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$

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3. Given $y = (2x+1)^3(3x-5)^5$,

a.) determine the derivative and then write your answer in simplified form.

8 marks

$\frac{dy}{dx} = 3(2)(2x+1)^2 \cdot (3x-5)^5 + (5)(3)(3x-5)^4 \cdot (2x+1)^3$ ✓

$\frac{dy}{dx} = 6(2x+1)^2(3x-5)^5 + 15(3x-5)^4 \cdot (2x+1)^3$ ✓

$\frac{dy}{dx} = 3(2x+1)^2(3x-5)^4 \cdot (2(3x-5) + 5(2x+1))$ ✓ $\rightarrow$ $\frac{dy}{dx} = 3(2x+1)^2(3x-5)(16x-5)$ ✓

b.) determine the value(s) of x for which the graph of $f(x)$ has a horizontal tangent.

$\boxed{\frac{dy}{dx} = 0}$ ✓ $\therefore 0 = 3(2x+1)^2(3x-5)(16x-5)$

$x = -\frac{1}{2}$ ✓ $x = \frac{5}{3}$ ✓ $x = \frac{5}{16}$ ✓

4. If $y = 4u^2 - 3u,\ u = x^2 - 1, evaluate\ \frac{dy}{dx}, for\ x = 2$

$\frac{dy}{du} = 8u - 3$ ✓ $\therefore \frac{dy}{du} \times \frac{du}{dx} = \frac{dy}{dx}$ ✓

$= (8u-3)(2x)$

$\frac{du}{dx} = 2x$ ✓ $= [8(x^2-1)-3](2x)$ ✓

at $x = 2$

$[8(2)^2-1)-3](2(2))$ ✓

$= (21)(4)$

$= 84$ ✓

6 marks

5. For the equation $y = 2(4x^3 - 5)^2$, predict with a formula,

a) the derivative that first becomes a constant and the value of the constant.

$y = 2(4x^3-5)(4x^3-5)$

$y = 32x^6 - 80x^3 + 50$ ✓

$\therefore a = 32$ ✓ $n = 6$ ✓

$\frac{d^6y}{dx^6} = (32)(6!)$ ✓

$= 23040$ ✓

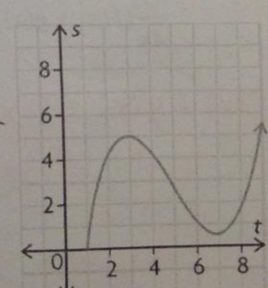
6 marks

b.) the value of the 8^{th} derivative.

$\frac{d^8y}{dx^8} = 0$ ✓

6. Given the position versus time graph determine

4 marks



a.) When the object stationary?

at $t = 3$ ✓, and $t = 7$ ✓

b.) When the object's velocity positive?

$1 < t < 3$ ✓ and $t > 7$ ✓

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APPLICATION

7. The tangent line to the curve defined by $f(x) = x^3 - 6x^2 + 8x$ at the point $(3,-3)$, intersects the curve at another point B. Find the coordinates of B.

x y

$f'(x) = 3x^2 - 12x + 8$ ✓

slope of tangent

at $x = 3$

$f'(3) = 3(3)^2 - 12(3) + 8$

$= -1$ ✓ slope of tangent

$m = -1$
$x = 3$ ✓
$y = -3$
$b = ?$

$y = mx + b$

$(-3) = (-1)(3) + b$

$0 = b$ ✓

$\therefore y = -x$ ✓

$\therefore -x = x^3 - 6x^2 + 8x$ ✓

$0 = x^3 - 6x^2 + 8x + x$

$0 = x(x^2 - 6x + 9)$

$0 = x(x-3)^2$ ✓

$x = 0$ ✓ $x = 3$ ✓

$f(0) = (0)^3 - 6(0) + 8(0)$

$= 0$ ✓

$\therefore (0,0)$ is point B

10 marks

8. Determine the equation or a possible equation for $f(x)$, and verify your work.

a.) $f'(x) = 6x - 4x^2 + 5$ and $f(-3) = -34$

possible

$f(x) = -\frac{4}{3}x^3 + 3x^2 + 5x - 82$ ✓

check

$f(-3) = -\frac{4}{3}(-3)^3 + 3(-3)^2 + 5(-3) - 84$ ✓

$= 36 + 27 - 15 - 82$

$= -34$ ✓

$f'(x) = 4x^2 + 6x + 5$ ✓

b.) $f'(x) = (5x+3)(6x) + (4 + 3x^2)(5)$

possible

$f(x) = (5x+3)(3x^2+4)$ ✓

check

$f'(x) = (5)(3x^2+4) + (6x)(5x+3)$ ✓

6 marks

9. Determine the equation of the tangent line to the graph of $y = -x^2 + 9x - 1$, where the slope of the tangent is 5.

$\frac{dy}{dx} = -2x + 9$ ✓

$\frac{dy}{dx} = 5$

$\therefore 5 = -2x + 9$ ✓

$2 = x$ ✓

at $x = 2$

$y = -(2)^2 + 9(2) - 1$

$y = 13$

$\therefore (x, y) = (2, 13)$ ✓

$m = 5$
$x = 2$ ✓
$y = 13$
$b = ?$

$y = mx + b$

$(13) = (5)(2) + b$ ✓

$3 = b$

$\therefore y = 5x + 3$ ✓

7 marks

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10. The position of an object moving along a straight line can be modeled by the function $s(t) = 3t^3 - 40.5t^2 + 162t$, where s is the position in metres at t seconds and $t \geq 0$.

a.) Determine the velocity and acceleration function for the object.

$s'(t) = v(t) = 9t^2 - 81t + 162$ ✓

$v'(t) = a(t) = 18t - 81$ ✓

11 marks

b.) Determine the position of the object when the velocity is 0.

$v(t) = 9t^2 - 81t + 162$

$\boxed{v(t) = 0}$

$0 = 9t^2 - 81t + 162$ ✓

$0 = 9(t^2 - 9 + 18)$

$0 = 9(t-6)(t-3)$ ✓

$t = 6$ ✓ & $t = 3$ ✓

$s(3) = 3(3)^3 - 40.5(3)^2 + 162(3)$

$= 202.5m$ ✓

$s(6) = 3(6)^3 - 40.5(6)^2 + 162(6)$

$= 162m$ ✓

c.) When does the object move to the left?

t	$s(t)$
0	0
1	124.5
2	186
3	202.5
4	192
5	172.5
6	162

t	$s(t)$
7	178.5
8	240
9	364.5 ✓
10	570
11	874.5
⋮	⋮

$\therefore 3 < t < 6$ ✓ the object moves left

d.) Is the object moving toward or away from its starting position when $t = 4$? Explain.

from part c.), the object is moving towards starting position at $t = 4$ ✓

11. Determine the value of "k" if

$$\frac{d\,(5x^3 - 6)^{-4}}{dx} = kx^2(5x^3 - 6)^{-5}$$

mistake

OMIT/IGNORE #11

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