MCV4U
April 6, 2016

KNOWLEDGE		
APPLICATION		
COMMUNICATION		

Name: ANSWERS

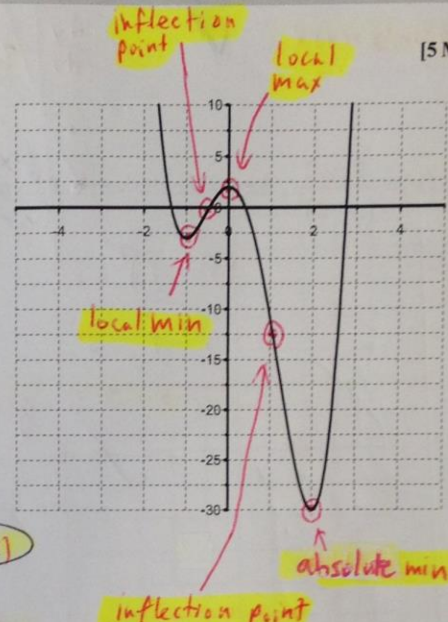
Parent/Guardian Signature: ______________

UNIT 3: CURVE SKETCHING & DERIVATIVES OF SINUSOIDAL FUNCTIONS

Communication reasoning, in writing and use of mathematical language, symbols and conventions will be assessed throughout this test.

KNOWLEDGE

1. The graph of the function $y = f(x)$ is shown at the right. **[5 MARKS]**



(a) Estimate the intervals where the function is decreasing.

$x < -1$

$0 < x < 2$ ✓

(b) Estimate the intervals where $f'(x) > 0$.

$-1 < x < 0$

$x > 2$ ✓

(c) Label the local maximum(s) and local minimum(s) ✓

(d) Label the inflection points ✓

(e) What is the value of $f''(x)$ at $x = 2$

mistake: not $f''(x)$

$f'(x) = 0$ ✓

2. Determine any horizontal ~~or oblique asymptotes~~ for each function. Also, determine if the function approaches the asymptote from above or below. oblique → bonus marks **[6 MARKS]**

(a) $h(x) = \dfrac{2x^2 - 3x - 2}{x - 1}$

above or below

$h(100) = 198.96$

$\therefore h(x)$ approaches from above as $x \to \infty$ ✓

$h(-100) = -200.97$

$\therefore h(x)$ approaches from below as $x \to -\infty$ ✓

horizontal asymptote

$$\lim_{x \to \infty} \frac{\frac{2x^2}{x} - \frac{3x}{x} - \frac{2}{x}}{\frac{x}{x} - \frac{1}{x}}$$

$= \frac{-3}{1}$ or -3 ✓

oblique asymptote $\boxed{2x - 1}$ Bonus

(b) $f(x)$ $y = \dfrac{x^2 - 4x + 1}{3x^2 - 1}$

above or below

$f(100) = 0.32004$

$\therefore f(x)$ approaches from below as $x \to \infty$ ✓

$f(-100) = 0.34671$

$\therefore f(x)$ approaches from above as $x \to -\infty$ ✓

horizontal asymptote

$$\lim_{x \to \infty} \frac{\frac{x^2}{x^2} - \frac{4x}{x^2} + \frac{1}{x^2}}{\frac{3x^2}{x^2} - \frac{1}{x^2}}$$

$= \frac{1}{3}$ ✓

oblique asymptote Bonus $\boxed{\text{None}}$

/11

3. Differentiate each of the following. Simplify your answer where possible. (Note: tanx = sinx/cosx) **[6 MARKS]**

(a) $y = \cos 3x$

$\frac{dy}{dx} = -3\sin 3x$ ✓

(b) $y = x\sin x$

$\frac{dy}{dx} = (1)\sin x + x\cos x$

$= \sin x + x\cos x$ ✓

(c) $y = \sin x^3$

$\frac{dy}{dx} = 3x^2\cos x^3$ ✓

(d) $y = \sin^3 x$

$\frac{dy}{dx} = 3\sin^2 x \cdot \cos x$ ✓

(e) $y = \cos(1 + x^3)$

$\frac{dy}{dx} = -3x^2\sin(1+x^3)$ ✓

(f) $y = 3\cos^2 x\tan^2 x$

$y = 3\cos^2 x\left(\frac{\sin^2 x}{\cos^2 x}\right)$

$y = 3\sin^2 x$

$y' = 6\sin x\cos x$ ✓

4. For $g(x) = \frac{3x-2}{(x-2)^2}$, determine the vertical asymptote(s) and describe its behavior on each side of the vertical asymptote. **[3 MARKS]**

$(x-2)^2 = 0$

$x - 2 = 0$

$x = 2$ ✓

$x \to 2^-$ (from the left)

$g(1.99) = \frac{3(1.99)-2}{(1.99-2)^2}$

$= \infty$ ✓

$\therefore$ as $x \to 2^-$, $g(x) \to \infty$

$x \to 2^+$ (from the right)

$g(2.01) = \frac{3(2.01)-2}{(2.01-2)^2}$

$= \infty$ ✓

as $x \to 2^+$, $g(x) \to \infty$

5. If $f(x) = \frac{4}{x^2-4}$, $f'(x) = \frac{-8x}{(x^2-4)^2}$ and $f''(x) = \frac{8(3x^2+4)}{(x^2-4)^3}$ determine

(a) the intervals of increase and decrease. **[3 MARKS]**

$f'(x) = 0$

$0 = \frac{-8x}{(x^2-4)^2}$

$0 = -8x$

$0 = x$ ✓

	$x < 0$	$x > 0$
$-8x$	+	−
$(x^2-4)^2$	+	+
sign of $f'(x)$	+	−
increase or decrease	increase	decrease

✓ ✓

(b) the intervals of concavity. **[3 MARKS]**

Second Derivative test

from part a.) $x = 0 \rightarrow$ critical number

$f''(0) = \frac{8(3(0)^2+4)}{((0)^2-4)^3}$ ✓

$= -0.5$ ✓

$f(0) = \frac{4}{(0)^2-4} = -1$

$\therefore$ Concave down ✓

max value at $(0, -1)$

/15

APPLICATION

6. Complete the tables below and use the information to sketch the graph of $f(x)$. **[5 MARKS]**

Using the function the following was determined:

- There are intercepts at $(x, y) = (2, 0), (-1, 0)$.
- *Vertical Asymptote*
  The line $x = 3$ is a vertical asymptote. $\therefore \lim_{x\to 3^-} f(x) = \infty$ and $\lim_{x\to 3^+} f(x) = \infty$
- *Horizontal Asymptote*
  $\lim_{x\to\pm\infty} f(x) = 2$

Using the first derivative the following was determined:

- The slope of the tangent is horizontal at $x = 0.5$.

	INTERVALS		
	$x < 0.5$	$0.5 < x < 3$	$x > 3$
$f'(x)$	(–)	(+)	(–)
$f(x)$	↘	↗	↘

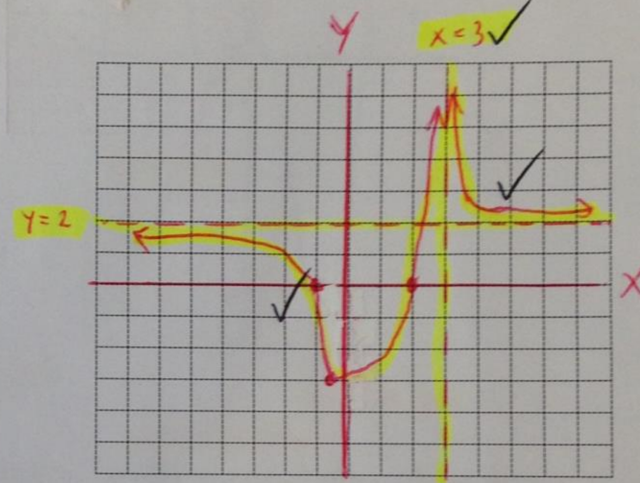
The function has a local minimum at $(x, y) = (0.5, -3)$.

Using the second derivative the following was determined:

- $f''(x) = 0$ at $x = -1$.

	INTERVALS		
	$x < -1$	$-1 < x < 3$	$x > 3$
$f''(x)$	(–)	(+)	(+)
$f(x)$	down	up	up

The function has an inflection point at $(x, y) = (-1, 0)$.



7. Determine the equation of the tangent to the graph of $y = x\sin 4x$ at $x = \frac{\pi}{4}$. **[5 MARKS]**

$\frac{dy}{dx} = (1)\sin 4x + 4x\cos 4x$

$\frac{dy}{dx} = \sin 4x + 4x\cos 4x$

sub $x = \frac{\pi}{4}$ into $\frac{dy}{dx}$

$\frac{dy}{dx} = \sin 4(\frac{\pi}{4}) + 4(\frac{\pi}{4})\cos 4(\frac{\pi}{4})$

$\frac{dy}{dx} = \sin\pi + \pi\cos\pi$

$\frac{dy}{dx} = -\pi$ ← slope of the tangent

sub $x = \frac{\pi}{4}$ into y

$y = \frac{\pi}{4}\sin 4(\frac{\pi}{4})$

$y = \frac{\pi}{4}(0)$

$y = 0$

$\therefore x = \frac{\pi}{4}$
$y = 0$
$m = -\pi$
$b = ?$

Note $\pi = 180°$
$\frac{\pi}{4} = 45°$

$y = mx + b$

$0 = (-\pi)(\frac{\pi}{4}) + b$

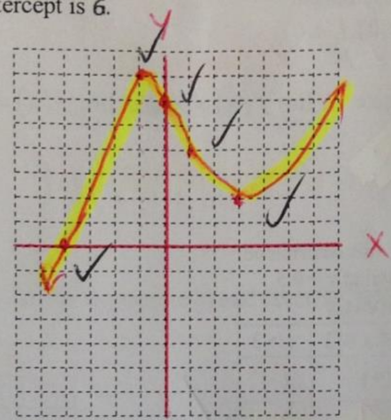
$\frac{\pi^2}{4} = b$

$\therefore y = -\pi x + \frac{\pi^2}{4}$

/10

8. Sketch a graph of a function with the following properties: **[5 MARKS]**

- There are local extrema at $(-1, 7)$ and $(3, 2)$.
- There is a point of inflection at $(1, 4)$.
- The graph is concave down only when $x < 1$.
- The x-intercept is -4, and the y-intercept is 6.



9. Determine the maximum and minimum values of the function $f(x) = \cos^2 x$ on the interval $[0, 2\pi]$ **[6 MARKS]**

$f'(x) = 2\cos x(-\sin x)$ ✓

Note $\sin 2x = 2\sin x \cos x$

$f'(x) = -2\sin x \cos x$

$f'(x) = -2\sin 2x$ ✓

Let $f'(x) = 0$ ✓

$0 = -2\sin 2x$

$0 = \sin 2x$ ✓

For $\sin x = 0$ at $0, \pi, 2\pi, 3\pi, 4\pi$

For $\sin 2x = 0$ at $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$ ✓

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
f(x)	1	0	1	0	1
max or min	max	min	max	min	max

/11