MCV4U
April 20, 2016

KNOWLEDGE		32
APPLICATION		30
COMMUNICATION		level

Name: ANSWERS

Parent/Guardian Signature: ____________________

UNIT 4: EXPONENTIAL FUNCTIONS & OPTIMIZATION PROBLEMS

Communication reasoning, in writing and use of mathematical language, symbols and conventions will be assessed throughout this test.

KNOWLEDGE

Out of 12 marks

1. Determine the derivative of each function

a. $f(x) = 7^x$

$f'(x) = 7^x \ln 7$ ✓

b. $g(x) = 7^{x^2}$

$g'(x) = 7^{x^2} \ln 7 \cdot 2x$ ✓

c. $h(x) = 5^{3x^2-2x}$

$h'(x) = 5^{3x^2-2x} \ln 5 \cdot (6x-2)$ ✓

d. $y = 3x\left(5^{x^2}\right)$

$\frac{dy}{dx} = (3)5^{x^2} + 5^{x^2}\ln 5 \cdot (2x)(3x)$ ✓

$\frac{dy}{dx} = 3 \cdot 5^{x^2}(1 + 2x^2 \ln 5)$ ✓

e. $y = xe^x$

$\frac{dy}{dx} = (1)e^x + xe^x$ ✓

$\frac{dy}{dx} = e^x(1+x)$ ✓

f. $y = e^{2x+1}$

$\frac{dy}{dx} = e^{2x+1} \cdot (2)$ ✓

f. $y = e^x - e^{-x}$

$\frac{dy}{dx} = e^x - e^{-x}(-1)$ ✓

$\frac{dy}{dx} = e^x + e^{-x}$ ✓

BONUS

g. $y = 2e^x \cos x$

$\frac{dy}{dx} = 2e^x \cos x + (-\sin x) \cdot 2e^x$ ✓

$\frac{dy}{dx} = 2e^x(\cos x - \sin x)$ ✓

h. $y = x^2 10^x$

$\frac{dy}{dx} = 2x \cdot 10^x + 10^x \ln 10 \cdot x^2$ ✓

$\frac{dy}{dx} = x10^x(2 + x\ln 10)$ ✓

2. Identify the local extrema of the function $f(x) = x^2e^x$

Out of 10 marks

$f'(x) = 2x \cdot e^x + x^2e^x$

$f'(x) = xe^x(x+2)$ ✓

Let $f'(x) = 0$

$x = 0$ ✓ $\quad e^x = 0$ ✓ $\quad x + 2 = 0$, $x = -2$ ✓

the range is $y > 0$ so there is no solution

$\therefore x = 0$ & $x = -2$

Intervals

	$x < -2$	$-2 < x < 0$	$x > 0$
x	$-$	$-$	$+$
e^x	$+$	$+$	$+$
$x+2$	$-$	$+$	$+$
sign of $f'(x)$	$+$	$-$	$+$
Increase or Decrease	↗	↘	↗

✓ ✓ ✓

$f(0) = (0)^2(e^0)$ ✓
$f(0) = 0$

$f(-2) = (-2)^2(e^{-2})$
$f(-2) = 4/e^2$

$(-2, \frac{4}{e^2})$ ✓ local max

$(0, 0)$ local min ✓

/22

3. Find the equation of the line tangent to the curve $y = 2e^x$ at $x = \ln 3$.

Find a point on the function

$y = 2e^x$

sub in $x = \ln 3$

$y = 2e^{\ln 3}$ ✓

$y = 2(3)$

$y = 6$ ✓

$\therefore (x, y) = (\ln 3, 6)$ ✓

$\frac{dy}{dx} = 2e^x$ ✓

sub in $x = \ln 3$

$y = 2e^{\ln 3}$ ✓

$y = 2(3)$

$y = 6$ ✓ ← slope of tangent

$y = 6$

$x = \ln 3$

$m = 6$ ✓

$b = ?$

$y = mx + b$

$(6) = (6)\ln 3 + b$ ✓

$6 - 6\ln 3 = b$ ✓

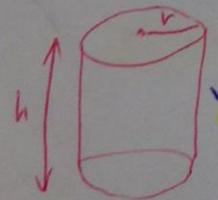
$\therefore y = 6x + 6 - 6\ln 3$ ✓

Out of 10 marks

/10

APPLICATION

4. What dimensions will maximize the volume of a cylinder if the total surface area of the cylinder is 150π cm^2?



Out of 10 marks

$V = \pi r^2 h$ ✓

$SA = 2\pi r^2 + 2\pi rh$

$150\pi = 2\pi r^2 + 2\pi rh$ ✓

$150\pi - 2\pi r^2 = 2\pi rh$

$\frac{150\pi - 2\pi r^2}{2\pi r} = h$

$\frac{75 - r^2}{r} = h$ ✓

$V = \pi r^2 h$

$V = \pi r^2 \left(\frac{75 - r^2}{r}\right)$

$V = \pi r(75 - r^2)$

$V = 75\pi r - \pi r^3$ ✓

$V' = 75\pi - 3\pi r^2$ ✓

Let $V' = 0$ ✓

$0 = 75\pi - 2\pi r^2$ ✓

$-75\pi = -\pi r^2$

$\sqrt{\frac{-75\pi}{-3\pi}} = r$

$5 = r$ ✓

$V(5) = 75\pi(5) - \pi(5)^3$ ✓

$V(6.12) = 785.4$ max

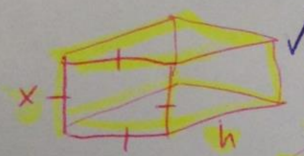
$h = \frac{75 - (5)^2}{5}$

$h = 10$

$\therefore$ dimensions of $r = 5$cm & $h = 10$cm will give a max volume of 785.4cm^3 ✓

/10

5. A closed rectangular storage box with a square base is to be constructed so the volume is 4000 cm^3. Determine the minimum cost of building the box if the material for the sides and top of the box costs \$4.50/m^2 and the material for the bottom of the box costs \$7/m^2.



$V = x^2h$

$0.004 = x^2h$

$\frac{0.004}{x^2} = h$

$SA = 4.5(2x^2+3xh) + 7(xh)$

$SA = 9x^2 + 13.5xh + 7xh$

$SA = 9x^2 + 20.5xh$

$SA = 9x^2 + 20.5x\left(\frac{0.004}{x^2}\right)$

$SA = 9x^2 + \frac{0.082}{x}$

$SA' = 18x - \frac{0.082}{x^2}$

$0 = 18x - \frac{0.082}{x^2}$

$\frac{-18x}{1} = \frac{-0.082}{x^2}$

$-18x^3 = -0.082$

$x^3 = 0.0455$

$x = \sqrt[3]{0.0455}$

$x = 0.166\text{ m} \rightarrow 16.6\text{ cm}$

$h = \frac{0.004}{(0.166)^2}$

$h = 0.145\text{ m} \rightarrow 14.5\text{ cm}$

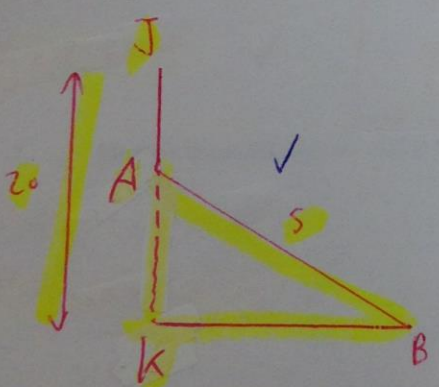
1 m = 100 cm
1 m^2 = 10000 cm^2
1 m^3 = 1000000 cm^3

$\therefore \frac{4000\text{ cm}^3}{1000000} = 0.004\text{ m}^3$

$\therefore$ the box will be 14.5 cm by 16.6 cm by 16.6 cm

Out of 10 marks

6. Jen and Kayla are both training for a marathon. Jen's house is located 20 km north of Kayla's house. At 9:00 one Saturday morning, Jen leaves her house and jogs south at 8 km/h. At the same time, Kayla leaves her house and jogs east at 6 km/h. When are Jen and Kayla closest together, given that they both run for 2.5 hours?



$JA = 8t$

$AK = 20 - 8t$

$KB = 6t$

$t \rightarrow$ time in hours
$s(t) \rightarrow$ distance as a function of time

Out of 10 marks

$s(t) = \sqrt{AK^2 + KB^2}$

$s(t) = \sqrt{(20-8t)^2 + (6t)^2}$

$s(t) = \sqrt{100t^2 - 320t + 400}$

$s(t) = (100t^2 - 320t + 400)^{\frac{1}{2}}$

$s'(t) = \frac{1}{2}(100t^2 - 320t + 400)^{-\frac{1}{2}} \cdot (200t - 320)$

$\frac{0}{1} = \frac{200t - 320}{2\sqrt{100t^2 - 320t + 400}}$

$0 = 200t - 320$

$1.6 = t$

1.6 hours
$= 1 + 0.6 \times 60$
$= 1$ hr 36 min

Domain $0 \le t \le 2.5$

$s(1.6) = 12 \rightarrow$ min

$s(0) = 20$

$s(2.5) = 15$

$\therefore$ minimum is 12 km which occurs at 10:36 am

/20