MCV4U
May 4, 2016

KNOWLEDGE		28
APPLICATION		28
COMMUNICATION		↑ level

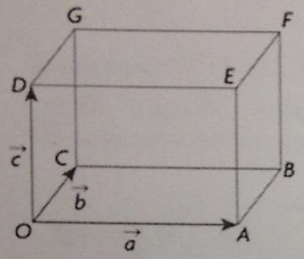
Name: ANSWERS

Parent/Guardian Signature: _____

UNIT 5: GEOMETRIC VECTORS

KNOWLEDGE

1. In the rectangular box shown below, $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OC} = \vec{b}$, and $\overrightarrow{OD} = \vec{c}$.



Out of 5 marks

Express each of the following vectors in terms of $\vec{a}$, $\vec{b}$, and $\vec{c}$.

a. $\overrightarrow{BC}$ b. $\overrightarrow{GF}$ c. $\overrightarrow{OB}$ d. $\overrightarrow{AC}$ e. $\overrightarrow{BG}$

$\overrightarrow{BC} = -\vec{a}$. ✓ $\overrightarrow{GF} = \vec{a}$. ✓ $\overrightarrow{OB} = \vec{a} + \vec{b}$. ✓ $\overrightarrow{AC} = \vec{b} - \vec{a}$. ✓ $\overrightarrow{BG} = \vec{c} - \vec{a}$. ✓

2. An airplane heads due south at a speed of 300 km/h and meets a wind from the west at 100 km/h. What is the resultant velocity of the airplane (relative to the ground)?

So, $|\vec{g}|^2 = |\vec{v} + \vec{w}|^2 = |\vec{v}|^2 + |\vec{w}|^2$

$|\vec{v} + \vec{w}|^2 = 300^2 + 100^2$ ✓

$|\vec{v} + \vec{w}|^2 = 100\,000$

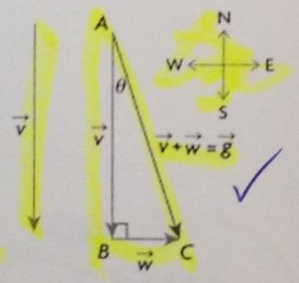
$|\vec{v} + \vec{w}| = \sqrt{100\,000}$

$\doteq 316.23$ ✓

$\tan\theta = \dfrac{|\vec{w}|}{|\vec{v}|} = \dfrac{100}{300} = \dfrac{1}{3}$ and $\theta = \tan^{-1}\left(\dfrac{1}{3}\right) \doteq 18.4°$ ✓

Out of 5 marks

This means that the airplane is heading S18.4°E at a speed of 316.23 km/h. ✓

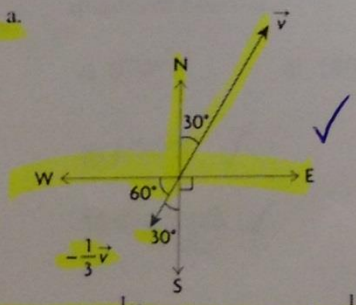


3. An airplane is flying in the direction $N30°E$ at an airspeed of 240 km/h. The velocity vector for this airplane is represented by $\vec{v}$.

a. Draw a sketch of $-\frac{1}{3}\vec{v}$ and state the direction of this vector.

b. For the vector $\frac{3}{2}\vec{v}$, state its direction and magnitude.

a.



b. The velocity vector $\frac{3}{2}\vec{v}$ represents a speed of $\frac{3}{2}(240 \text{ km/h}) = 360 \text{ km/h}$ in the same direction as $\vec{v}$. ✓

Out of 4 marks.

The vector $-\frac{1}{3}\vec{v}$ represents a speed of $\frac{1}{3}(240 \text{ km/h}) = 80 \text{ km/h}$ and points in the opposite direction as $\vec{v}$. The direction for this vector can be described as W60°S, S30°W, or a bearing of 210°. ✓

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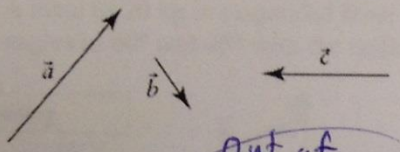
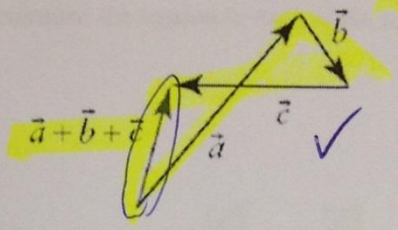
Use the following set of vectors to draw a diagram of each expression.

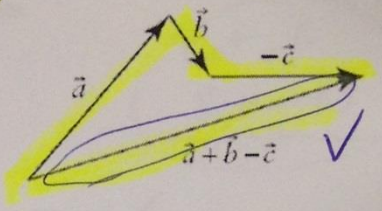
a) $\vec{a}+\vec{b}+\vec{c}$ b) $\vec{a}+\vec{b}-\vec{c}$

a)



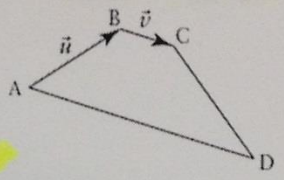
b)





Out of 2 marks

5. In trapezoid ABCD, BC ∥ AD and AD = 3BC. Let $\overrightarrow{AB}=\vec{u}$ and $\overrightarrow{BC}=\vec{v}$. Express $\overrightarrow{AD}$, $\overrightarrow{BD}$, and $\overrightarrow{CD}$ as linear combinations of $\vec{u}$ and $\vec{v}$.



$\overrightarrow{AD}=3\vec{v}$ ✓

$\overrightarrow{BD}=\overrightarrow{BA}+\overrightarrow{AD}$

$=-\vec{u}+3\vec{v}$ ✓

$\overrightarrow{CD}=\overrightarrow{CB}+\overrightarrow{BA}+\overrightarrow{AD}$

$=-\vec{v}+(-\vec{u})+3\vec{v}$

$=(-\vec{u})+(-\vec{v})+3\vec{v}$

$=-\vec{u}+2\vec{v}$ ✓

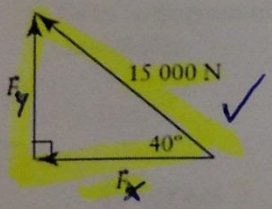
Out of 3 marks

6. A tow truck is pulling a car from a ditch. The tension in the cable is 15 000 N at an angle of 40° to the horizontal.

a) Draw a diagram showing the resolution of the force into its rectangular components.

b) Determine the magnitudes of the horizontal and vertical components of the force.

a) The tension can be resolved into two rectangular components: vertical, $\vec{F}_y$, and horizontal, $\vec{F}_x$



b) $\frac{|\vec{F}_x|}{\cos 40°}=15\,000$ ✓

$|\vec{F}_x|=15\,000\cos 40°$

$\doteq 11\,490$ ✓

$\frac{|\vec{F}_y|}{\sin 40°}=15\,000$ ✓

$|\vec{F}_y|=15\,000\sin 40°$

$\doteq 9642$ ✓

The magnitude of the horizontal component is about 11 500 N, and the magnitude of the vertical component is about 9600 N.

Out of 5 marks

7. Convert each true bearing to its equivalent quadrant bearing.

a) 070° b) 180°

a) N70°E ✓

b) due south ✓

8. Convert each quadrant bearing to its equivalent true bearing.

a) N35°E b) N70°W

a) 035° ✓

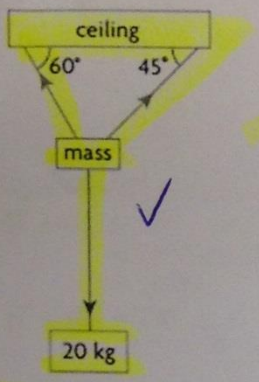
b) 290° ✓

Out of 4 marks

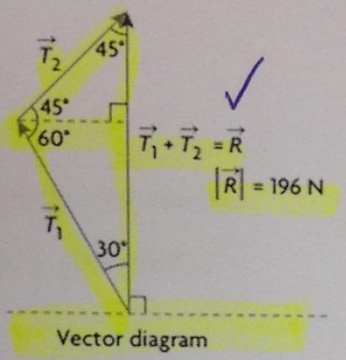
LICATION

. A mass of 20 kg is suspended from a ceiling by two lengths of rope that make angles of 60° and 45° with the ceiling. Determine the tension in each of the ropes.

Out of 8 marks



Position diagram



Vector diagram

$$\frac{|\vec{T}_1|}{\sin 45°} = \frac{|\vec{T}_2|}{\sin 30°} = \frac{196}{\sin 105°}$$

$|\vec{T}_1|\sin 105° = 196(\sin 45°)$ and $|\vec{T}_2|\sin 105° = 196(\sin 30°)$

$|\vec{T}_1| = \frac{196(0.7071)}{0.9659} \doteq 143.48$ N and $|\vec{T}_2| = \frac{196(0.5)}{0.9659} \doteq 101.46$ N

Therefore, the tensions in the two ropes are approximately 143.48 N and 101.46 N.

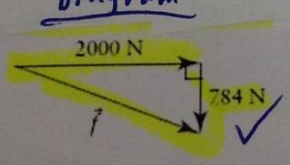
9. A clown with mass 80 kg is shot out of a cannon with a horizontal force of 2000 N. The vertical force is the acceleration due to gravity, which is 9.8 m/s^2, times the mass of the clown.

b.) Find the magnitude and direction of the equilibriant force on the clown

a) Find the magnitude and direction of the resultant force on the clown.

a) Draw a diagram of the situation. The vertical force is 9.8×80 or 784 N downward.

Diagram



$|\vec{f}|^2 = 2000^2 + 784^2$

$= 4\,614\,656$

$|\vec{f}| \doteq 2148$

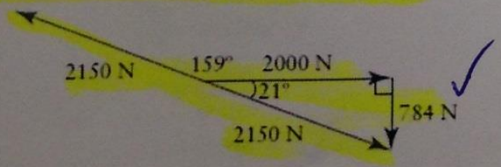
$\tan\theta = \frac{784}{2000}$

$\theta = \tan^{-1}\frac{784}{2000}$

$\doteq 21°$

The resultant has a magnitude of about 2150 N and a direction of 21° below the horizontal.

b) Draw the equilibrant on the diagram.



Out of 8 marks

The equilibrant has magnitude of about 2150 N and a direction of 159° counterclockwise from the horizontal.

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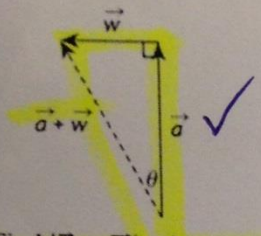
A small airplane is flying due north at 150 km/h when it encounters a wind of 80 km/h from the east. What is the resultant ground velocity of the airplane?



Find $|\vec{a} + \vec{w}|$ using the Pythagorean theorem.

$|\vec{a} + \vec{w}|^2 = |\vec{a}|^2 + |\vec{w}|^2$

$= (150 \text{ km/h})^2 + (80 \text{ km/h})^2$

$= 28900$

$|\vec{a} + \vec{w}|^2 = 170$

Find the direction of $\vec{a} + \vec{w}$ using the ratio

$$\tan(\theta) = \frac{|\vec{w}|}{|\vec{a}|}$$

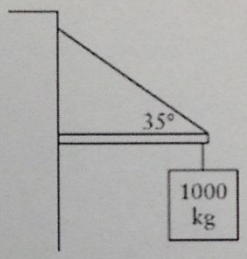
$$\theta = \tan^{-1} \frac{80 \text{ km/h}}{150 \text{ km/h}}$$

$\doteq$ N 28.1° W

$\vec{a} + \vec{w} = 170$ km/h, N 28.1° W

Out of 6 marks

11. A 1000-kg load is suspended from the end of a horizontal boom. The boom is supported by a cable that makes an angle of 35° with the boom.



a) What is the weight of the load?

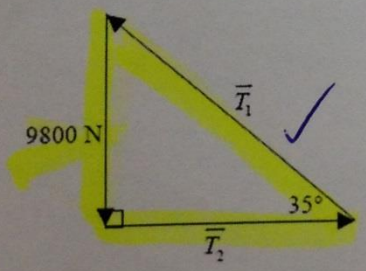
b) What is the tension in the cable?

c) What is the horizontal force on the boom?

d) What is the vertical equilibrant component of the tension in the cable?

a) The weight of the load is the mass times the force of gravity: 1000(9.8) or 9800 N.

b) There are three forces present: the weight, tension in the boom, and tension in the cable. These three forces form a right-angled triangle.



$$\frac{|\vec{T}_1|}{9800} = \csc 35°$$

$$|\vec{T}_1| = 9800 \csc 35°$$

$$= \frac{9800}{\sin 35°}$$

$\doteq 17\,085.8$

c.)

$$\frac{|\vec{T}_2|}{9800} = \cot 35°$$

$$|\vec{T}_2| = 9800 \cot 35°$$

$$= \frac{9800}{\tan 35°}$$

$\doteq 13\,995.9$

Out of 6 marks

d) The vertical equilibrant component of the tension in the cable is −9800 N, the opposite of the weight of the sign.

12.