MCV4U
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Name: ANSWERS

UNIT 6: CARTESIAN VECTORS

KNOWLEDGE

1. State whether each quantity is a vector or a scalar.

(a) $\vec{a}+\vec{b}+\vec{c}$ vector ✓ (b) $(\vec{a}\cdot\vec{b})\vec{b}$ vector ✓ (c) $|\vec{b}\downarrow\vec{a}|$ scalar ✓ (d) $k\vec{b}$ vector ✓

Out of 4

2. Given $\vec{u}=[-2,4]$ and $\vec{v}=[3,2]$, determine the angle between the given vectors.

$|\vec{u}| = \sqrt{(-2)^2+(4)^2}$
$= \sqrt{20}$ ✓

$|\vec{v}| = \sqrt{(3)^2+(2)^2}$
$= \sqrt{13}$ ✓

$\vec{u}\cdot\vec{v} = (-2)(3)+(4)(2)$
$= 2$ ✓

$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}$ ✓

$\angle\theta = \cos^{-1}\left(\frac{2}{\sqrt{20}\sqrt{13}}\right)$ ✓

$\angle\theta = 83°$ ✓

Out of 6

3. If $\vec{a}=[6,1,-4]$ and $\vec{b}=[1,-1,2]$, determine

(a) $\vec{a}\cdot\vec{b}$

$= (6)(1)+(1)(-1)+(-4)(2)$ ✓
$= -3$ ✓

(b) $\vec{a}\times\vec{b}$

$= (a_2b_3-a_3b_2, a_3b_1-a_1b_3, a_1b_2-a_2b_1)$
$= [(1)(2)-(-4)(-1), (-4)(1)-(6)(2), (6)(-1)-(1)(1)]$ ✓
$= (-2,-16,-7)$ ✓

Out of 8

(c) $|\vec{a}+\vec{b}|$

$\vec{a}+\vec{b} = [(6)+(1), (1)+(-1), (-4)+(2)]$ ✓
$= (7,0,-2)$ ✓

$|\vec{a}+\vec{b}| = \sqrt{(7)^2+(0)^2+(-2)^2}$ ✓
$= \sqrt{53}$ ✓

4. If $\vec{m}=-\vec{i}+2\vec{j}+\vec{k}$ and $\vec{n}=3\vec{i}-2\vec{j}+\vec{k}$, determine $3\vec{m}+\vec{n}$.

$3\vec{m} = 3(-\vec{i}+2\vec{j}+\vec{k})$ ✓
$= -3\vec{i}+6\vec{j}+3\vec{k}$ ✓

$3\vec{m}+\vec{n} = (-3\vec{i}+6\vec{j}+3\vec{k})+(3\vec{i}-2\vec{j}+\vec{k})$ ✓
$= 4\vec{j}+4\vec{k}$ ✓

Out of 4

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5. Given $A(5,7)$ and $B(-3,12)$, determine

(a) $\overline{AB}$

$= (-3-5, 12-7)$ ✓

$= (-8,5)$ ✓

(b) $|\overline{AB}|$

$= \sqrt{(-8)^2+(5)^2}$ ✓

$= \sqrt{89}$ ✓

Out of 4

6. Given $\vec{a} = (-7,21,14)$ and $\vec{b} = (6,-18,p)$, determine the value of p such that $\vec{a}$ and $\vec{b}$ are

(a) perpendicular to each other.

$\vec{a}\cdot\vec{b} = (-7)(6)+(21)(-18)+(14)(p)$ ✓

$\vec{a}\cdot\vec{b} = -420+14p$ ✓

Let $\vec{a}\cdot\vec{b} = 0$ ✓

$0 = -420+14p$

$30 = p$ ✓

(b) collinear.

$\vec{a} = k\vec{b}$ ✓

$(-7,21,14) = k(6,-18,p)$ ✓

$-7 = 6k$ $\quad 21 = -18k$ $\quad 14 = kp$

$-\frac{7}{6} = k$ ✓ $\quad -\frac{7}{6} = k$ ✓ $\quad 14 = \left(-\frac{7}{6}\right)p$ ✓

$-12 = p$ ✓

Out of 10

7. For each vector below,

i) sketch the position vector

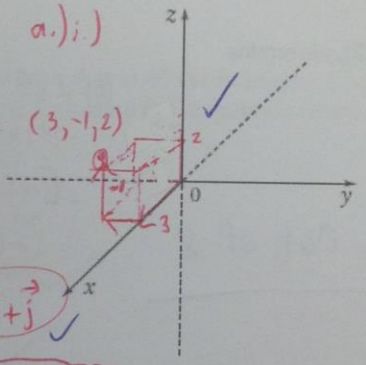
ii) write the vector in the form $a\vec{i}+b\vec{j}+c\vec{k}$

iii) find the magnitude

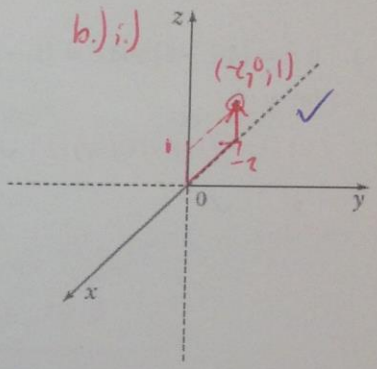
a) $\vec{u} = [3,-1,2]$

b) $\vec{v} = [-2,0,1]$

a.)i.)



b.)i.)



a)ii) $\vec{u} = 3\vec{i}-\vec{j}+2\vec{k}$ ✓

a)iii) $|\vec{u}| = \sqrt{(3)^2+(-1)^2+(2)^2}$ ✓

$|\vec{u}| = \sqrt{14}$ ✓

b)ii) $\vec{v} = -2\vec{i}+\vec{j}$ ✓

b)iii) $|\vec{v}| = \sqrt{(2)^2+(0)^2+(1)^2}$ ✓

$|\vec{v}| = \sqrt{5}$ ✓

Out of 8

8. Suppose $\vec{u} = [4,2]$ and $\vec{v} = [5,-1]$.

(a) Determine $\vec{u} \downarrow \vec{v}$

$\vec{u}\cdot\vec{v} = (4)(5)+(2)(-1)$

$= 18$ ✓

$|\vec{v}| = \sqrt{(5)^2+(-1)^2}$

$= \sqrt{26}$ ✓

$|\vec{v}|^2 = (\sqrt{26})^2$

$= 26$ ✓

$\vec{u}\downarrow\vec{v} = \left(\frac{\vec{u}\cdot\vec{v}}{|\vec{v}|^2}\right)\vec{v}$ ✓

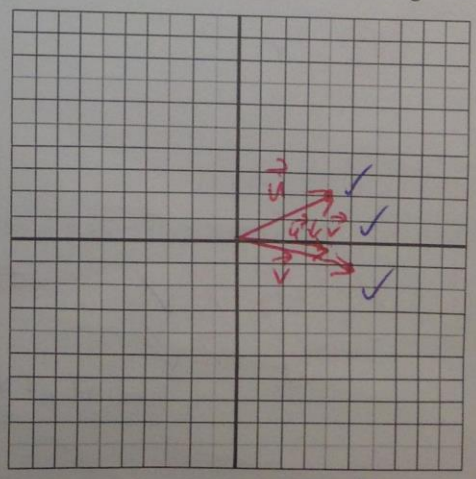
$= \left(\frac{18}{26}\right)(5,-1)$ ✓

$= \left(\frac{45}{13}, -\frac{9}{13}\right)$

$= (3.5,-0.7)$ ✓

Out of 9

(b) Graph vector of $\vec{u}$, $\vec{v}$ and $\vec{u} \downarrow \vec{v}$, on the grid below.



✓ ✓ ✓

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APPLICATION

9. Vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 5$, $|\vec{b}| = 3$ and $|\vec{a} + \vec{b}| = 6$. Determine $2\vec{a} \cdot \vec{b}$

$\vec{c} = \vec{a} + \vec{b}$

$|\vec{c}| = |\vec{a} + \vec{b}|$ ✓

∴ cosine law ✓

$|c|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos\theta$ ✓

$|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos\theta$ ✓

$(6)^2 = (5)^2 + (3)^2 - 2(5)(3)\cos\theta$ ✓

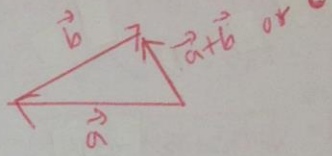
$\cos\theta = \frac{-2}{30}$ ✓

$\cos\theta = \frac{-1}{15}$ ✓

$2\vec{a} \cdot \vec{b} = 2|\vec{a}||\vec{b}|\cos\theta$ ✓

$2\vec{a} \cdot \vec{b} = 2(5)(3)\left(\frac{-1}{15}\right)$ ✓

$2\vec{a} \cdot \vec{b} = -2$ ✓



Out of 10

10. If the vectors $\vec{a} + 3\vec{b}$ and $4\vec{a} - \vec{b}$ are perpendicular, and $|\vec{a}| = 2|\vec{b}|$, determine the angle (to the nearest degree) between the nonzero vectors $\vec{a}$ and $\vec{b}$.

Note: $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$(\vec{a} + 3\vec{b}) \cdot (4\vec{a} - \vec{b}) = 0$ ✓

$\vec{a} \cdot (4\vec{a} - \vec{b}) + 3\vec{b}(4\vec{a} - \vec{b}) = 0$ ✓

$4\vec{a} \cdot \vec{a} - \vec{a} \cdot \vec{b} + 12\vec{a} \cdot \vec{b} - 3\vec{b} \cdot \vec{b} = 0$ ✓

$4|\vec{a}|^2 + 11\vec{a} \cdot \vec{b} - 3|\vec{b}|^2 = 0$ ✓

$4(2|\vec{b}|)^2 + 11(|\vec{a}||\vec{b}|\cos\theta) - 3|\vec{b}|^2 = 0$ ✓

$16|\vec{b}|^2 + 11(2|\vec{b}|)|\vec{b}|\cos\theta - 3|\vec{b}|^2 = 0$ ✓

$16|\vec{b}|^2 + 22|\vec{b}|^2\cos\theta - 3|\vec{b}|^2 = 0$ ✓

$22|\vec{b}|^2\cos\theta = -13|\vec{b}|^2$

$\cos\theta = \frac{-13|\vec{b}|^2}{22|\vec{b}|^2}$ ✓

$\theta = \cos^{-1}\left(\frac{-13}{22}\right)$ ✓

$\theta = 126.2°$ ✓

Out of 10

11. Marianna is pulling her daughter in a toboggan and is exerting a force of 40 N, acting at 24° to the ground. If Marianna pulls the child a distance of 100 m, how much work was done?

$W = |\vec{f}||\vec{d}|\cos\theta$ ✓

$W = (40)(100)\cos 24°$ ✓

$W = 3654.2\text{ J}$ ✓

Out of 4

∴ about 3654.2 J of work is done ✓ to pull the toboggan

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12. Using vectors, calculate the area of a triangle with vertices $A(-1, 3, 5)$, $B(2, 1, 3)$ and $C(-1, 1, 4)$.

$\vec{AB} = (2-(-1), 1-3, 3-5)$

$\vec{AB} = (3, -2, -2)$ ✓ $\rightarrow \vec{a}$

$\vec{AC} = (-1-(-1), 1-3, 4-5)$

$\vec{AC} = (0, -2, -1)$ ✓ $\rightarrow \vec{b}$

Area of a triangle $= \frac{|\vec{a} \times \vec{b}|}{2}$ ✓

$\vec{a} \times \vec{b} = [(-2)(-1)-(-2)(-2), (-2)(0)-(3)(-1), (3)(-2)-(-2)(0)]$ ✓

$= (-2, 3, -4)$ ✓

$|\vec{a} \times \vec{b}| = \sqrt{(-2)^2+(3)^2+(-4)^2}$

$|\vec{a} \times \vec{b}| = \sqrt{29}$ ✓

$= \frac{\sqrt{29}}{2}$ ✓

$= 5,39 \text{ units}^2$ ✓

Out of 8

13. A 20 N force is applied at the end of a wrench that is 40 cm in length. The force is applied at an angle of 60° to the wrench. Calculate the magnitude of the torque about the point of rotation M.

$0.4\text{m} = 40\text{cm}$ ✓

$T = |\vec{r}||\vec{f}| \sin\theta$ ✓

$T = (0.4)(20) \sin 60°$ ✓

$T = 6,92\text{J}$ ✓

∴ torque is about 6.92 J ✓

Out of 5

14. Find the volume of the parallelepiped, defined by the vectors $\vec{u}$, $\vec{v}$, and $\vec{w}$.
$\vec{u} = [1, 1, 9]$, $\vec{v} = [0, 0, 4]$, and $\vec{w} = [-2, 0, 5]$

$\vec{u} \times \vec{v} = [(1)(4)-(9)(0), (9)(0)-(1)(4), (1)(0)-(1)(0)]$ ✓

$\vec{u} \times \vec{v} = (4, -4, 0)$ ✓

volume of a parallelepiped $= |\vec{w} \cdot (\vec{u} \times \vec{v})|$ ✓

$= |(-2)(4)+(0)(-4)+(5)(0)|$ ✓

$= 8 \text{ units}^3$ ✓

Out of 5