

KNOWLEDGE		26
APPLICATION		28
COMMUNICATION		LEVEL

Name: ANSWERS

Parent/Guardian Signature: _____

UNIT 7: LINES & PLANES

Communication reasoning, in writing and use of mathematical language, symbols and conventions will be assessed throughout this test.

KNOWLEDGE

1. Determine each of the following:

(a) The vector equation of the line passing through $A(0, -3, 7)$ and $B(1, 2, -1)$. [2 MARKS]

$$\vec{AB} = (1-0, 2-(-3), -1-7)$$

$$= (1, 5, -8) \quad \checkmark$$

$$\therefore [x, y, z] = (0, -3, 7) + s(1, 5, -8) \quad \checkmark$$

(b) The parametric equations of the line $l: (x, y, z) = (0, 2, -1) + t(4, -1, 1)$. [3 MARKS]

$$\begin{aligned} x &= 4t \quad \checkmark \\ y &= 2 - t \quad \checkmark \\ z &= -1 + t \quad \checkmark \end{aligned}$$

(c) The vector equation of the plane passing through $A(2, -4, 3)$, $B(2, 1, 0)$ and $C(0, 0, 1)$. [3 MARKS]

$$\vec{AB} = (2-2, 1-(-4), 0-3)$$

$$= (0, 5, -3) \quad \checkmark$$

$$[x, y, z] = (2, -4, 3) + s(0, 5, -3) + t(-2, 4, -2) \quad \checkmark$$

$$\vec{AC} = (0-2, 0-(-4), 1-3)$$

$$= (-2, 4, -2) \quad \checkmark$$

(d) The scalar (Cartesian) of the plane $\pi: (x, y, z) = (0, 2, 1) + s(1, -1, 2) + t(3, 0, -2)$. [3 MARKS]

$$\vec{m}_1 = (1, -1, 2)$$

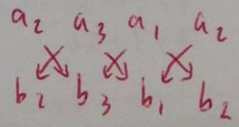
$$\vec{m}_2 = (3, 0, -2)$$

$$\vec{n} = \vec{m}_1 \times \vec{m}_2$$

$$= (a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1)$$

$$= [(-1)(-2) - (2)(0), (2)(3) - (1)(-2), (1)(0) - (-1)(3)]$$

$$= (2, 8, 4) \quad \checkmark$$



$$AX + BY + CZ + D = 0$$

$$(2)(0) + (8)(2) + (4)(1) + D = 0$$

$$D = -20 \quad \checkmark$$

$$\therefore 2X + 8Y + 4Z - 20 = 0 \quad \checkmark$$

2. Determine the point of intersection between the following pair of lines:

[5 MARKS]

$$l_1: (x, y, z) = (7, 2, -6) + s(2, 1, -3)$$

$$l_2: (x, y, z) = (3, 9, 13) + t(1, 5, 5)$$

$$\begin{aligned} L_1: & x = 7 + 2s \\ & y = 2 + s \\ & z = -6 - 3s \\ L_2: & x = 3 + t \\ & y = 9 + 5t \\ & z = 13 + 5t \end{aligned}$$

Let x's equal each other for L1 & L2

$$7 + 2s = 3 + t$$

$$\textcircled{1} t = 4 + 2s$$

Let y's equal each other for L1 & L2

$$2 + s = 9 + 5t$$

$$\textcircled{2} t = \frac{-7 + s}{5}$$

Let ① equal ②

$$4 + 2s = \frac{-7 + s}{5}$$

$$20 + 10s = -7 + s$$

$$9s = -27$$

$$s = -3$$

sub $s = -3$ into L1

$$(x, y, z) = (7, 2, -6) + (-3)(2, 1, -3)$$

$$= (1, -1, 3)$$

Point of intersection

3. Determine the equation of the line of intersection between the planes

[5 MARKS]

$$\pi_1: 2x + 3y - z - 3 = 0 \text{ and } \pi_2: -x + y + z - 1 = 0$$

$$\textcircled{1} 2x + 3y - z = 3$$

$$\textcircled{2} -x + y + z = 1$$

$$x + 4y = 4$$

Let $y = s$

$$x + 4s = 4$$

$$x = 4 - 4s$$

$$y = s$$

sub x & y into π_1

$$2(4 - 4s) + 3(s) - z - 3 = 0$$

$$8 - 8s + 3s - z - 3 = 0$$

$$-z = -5 + 5s$$

$$z = 5 - 5s$$

line of intersection

$$\therefore x = 4 - 4s$$

$$y = s$$

$$z = 5 - 5s$$

4. Determine the point of intersection between the line $l: (x, y, z) = (2, -5, 3) + t(3, 1, 5)$ and the plane $\pi: 5x + y - 2z + 19 = 0$.

[5 MARKS]

$$\begin{aligned} L: & x = 2 + 3t \\ & y = -5 + t \\ & z = 3 + 5t \end{aligned}$$

sub L into π

$$5(2 + 3t) + (-5 + t) - 2(3 + 5t) + 19 = 0$$

$$10 + 15t - 5 + t - 6 - 10t + 19 = 0$$

$$6t = -18$$

$$t = -3$$

sub $t = -3$ into L

$$x = 2 + 3(-3)$$

$$= -7$$

$$y = -5 + (-3)$$

$$= -8$$

$$z = 3 + 5(-3)$$

$$= -12$$

Point of intersection

$$\therefore (-7, -8, -12)$$

APPLICATION

5. Determine the vector equation of the line that passes through the point $P(-1, 2, 5)$ and is perpendicular to the plane $3x - 4y + 2z - 7 = 0$. [3 MARKS]

$\vec{n} = (3, -4, 2)$

$(x, y, z) \cdot (3, -4, 2) = 0$

$3x - 4y + 2z = 0$

let $x = 1$ & $y = 1$

$3(1) - 4(1) + 2z = 0$

$z = \frac{1}{2}$

$\therefore [x, y, z] = (-1, 2, 5) + s(1, 1, \frac{1}{2})$

$\therefore \vec{m} = (1, 1, \frac{1}{2})$

6. In each case, state whether the system has no solutions, a unique solution (ie point of intersection), or an infinite number of solutions (ie line or plane of solutions). Explain. Do not solve the system! [9 MARKS]

(a) ① $2x - y + 3z = 5$

② $4x + y + z = -2$

③ $3x - y + z = 7$

$\vec{n}_1 = (2, -1, 3)$

$\vec{n}_2 = (4, 1, 1)$

$\vec{n}_3 = (3, -1, 1)$

normals not parallel or coincident

④ & ⑤ are not parallel or coincident so there is a point of intersection

① $2x - y + 3z = 5$

3x ② $12x + 3y + 3z = -6$

④ $-10x - 4y = 11$

② $4x + y + z = -2$

③ $3x - y + z = 7$

⑤ $x + 2y = -9$

(b) ① $2x - y + z = 1$

② $3x - 5y + 4z = 3$

③ $3x + 2y - z = -1$

$\vec{n}_1 = (2, -1, 1)$

$\vec{n}_2 = (3, -5, 4)$

$\vec{n}_3 = (3, 2, -1)$

normals not parallel or coincident

④ & ⑤ are parallel so there is no solution

① $2x - y + z = 1$

③ $3x + 2y - z = -1$

④ $5x + y = 0$

4x ① $\rightarrow 8x - 4y + 4z = 4$

③ $\rightarrow 3x - 5y + 4z = 3$

⑤ $5x + y = 1$

(c) ① $3x - 9y + 3z = 9$

② $x - 3y + z = 3$

③ $2x - 6y + 2z = 12$

$\vec{n}_1 = (3, -9, 3)$

$\vec{n}_2 = (1, -3, 1)$

$\vec{n}_3 = (2, -6, 2)$

\therefore ①, ② & ③ are all coincident

infinite solutions

$3 \times ② = ① \rightarrow 3 \times ② \quad 3x - 9y + 3z = 9$
 $① \quad 3x - 9y + 3z = 9$ } coincident

$1.5 \times ③ = ① \rightarrow 1.5 \times ③ \quad 3x - 9y + 3z = 9$
 $① \quad 3x - 9y + 3z = 9$ } coincident

7. Consider the following system of equations:

① $2x + y + 6z = p$

② $x + my + 3z = q$

Determine the values of m, p and q such that the two planes.

(a) are coincident. [3 MARKS]

① $(2, 1, 6)$

② $(1, m, 3)$

$2 \times \text{①} = \text{②}$

$2(1, m, 3) = (2, 1, 6)$

$(2, 2m, 6) = (2, 1, 6)$

$2m = 1$
 $m = \frac{1}{2}$

(b) intersect at right angles. [3 MARKS]

$(2, 1, 6) \cdot (1, m, 3) = 0$

$2 + m + 18 = 0$

$m = -20$

8. Determine the solution to the following system of equations: [10 MARKS]

① $x - y + z = 1$

② $x + y + 2z = 2$

③ $x - 5y - z = 1$

$\vec{n}_1 = (1, -1, 1)$
 $\vec{n}_2 = (1, 1, 2)$
 $\vec{n}_3 = (1, -5, -1)$

not parallel or coincident

$-1 \times \text{①} \rightarrow -x + y - z = -1$

$\oplus \quad x + y + 2z = 2$

④ $2y + z = 1$

$\vec{m}_1 = \vec{n}_1 \times \vec{n}_2$

$= [(-1)(2) - (1)(1), (1)(1) - (1)(2), (1)(1) - (-1)(1)]$

$= (-3, -1, 2)$

$-1 \times \text{①} \rightarrow -x + y - z = -1$

③ $\rightarrow x - 5y - z = 1$

⑤ $-4y - 2z = 0$

$\vec{m}_2 = \vec{n}_1 \times \vec{n}_3$

$= [(-1)(-1) - (1)(-5), (1)(1) - (1)(-1), (1)(-5) - (-1)(1)]$

$= (6, 2, -4)$

$= -2(-3, -1, 2)$

$2 \times \text{④} \rightarrow 4y + 2z = 2$

⑤ $\rightarrow -4y - 2z = 0$

$0y + 0z = 2$

no solution

$\vec{m}_3 = \vec{n}_2 \times \vec{n}_3$

$= [(1)(-1) - (2)(-5), (2)(1) - (1)(-1), (1)(-5) - (1)(1)]$

$= (9, 3, -6)$

$= -3(-3, -1, 2)$

\therefore the systems of equations are inconsistent and form a triangular prism.