MCV4U
June 14, 2016

KNOWLEDGE		26
APPLICATION		28
COMMUNICATION		LEVEL

Name: ANSWERS

Parent/Guardian Signature: ____________

UNIT 7: LINES & PLANES

Communication reasoning, in writing and use of mathematical language, symbols and conventions will be assessed throughout this test.

KNOWLEDGE

1. Determine each of the following:

(a) The vector equation of the line passing through $A(0,-3,7)$ and $B(1,2,-1)$. [2 MARKS]

$\vec{AB} = (1-0, 2-(-3), -1-7)$
$= (1,5,-8)$ ✓

$\therefore [x,y,z] = (0,-3,7) + s(1,5,-8)$ ✓

(b) The parametric equations of the line $l: (x,y,z) = (0,2,-1) + t(4,-1,1)$. [3 MARKS]

$x = 4t$ ✓
$y = 2 - t$ ✓
$z = -1 + t$ ✓

(c) The vector equation of the plane passing through $A(2,-4,3)$, $B(2,1,0)$ and $C(0,0,1)$. [3 MARKS]

$\vec{AB} = (2-2, 1-(-4), 0-3)$
$= (0,5,-3)$ ✓

$\vec{AC} = (0-2, 0-(-4), 1-3)$
$= (-2,4,-2)$ ✓

$[x,y,z] = (2,-4,3) + s(0,5,-3) + t(-2,4,-2)$ ✓

(d) The scalar (Cartesian) of the plane $\pi: (x,y,z) = \underset{x\ y\ z}{(0,2,1)} + s\underset{a_1\ a_2\ a_3}{(1,-1,2)} + t\underset{b_1\ b_2\ b_3}{(3,0,-2)}$. [3 MARKS]

$\vec{m_1} = (1,-1,2)$
$\vec{m_2} = (3,0,-2)$
$\vec{n} = \vec{m_1} \times \vec{m_2}$
$= (a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1)$
$= [(-1)(-2)-(2)(0), (2)(3)-(1)(-2), (1)(0)-(-1)(3)]$
$= (2,8,3)$ ✓

$a_2\ a_3\ a_1\ a_2$
$b_2\ b_3\ b_1\ b_2$

$Ax + By + Cz + D = 0$
$(2)(0) + (8)(2) + (3)(1) + D = 0$
$D = -19$ ✓

$\therefore 2x + 8y + 3z - 19 = 0$ ✓

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2. Determine the point of intersection between the following pair of lines: [5 MARKS]
$\ell_1: (x, y, z) = (7, 2, -6) + s(2, 1, -3)$
$\ell_2: (x, y, z) = (3, 9, 13) + t(1, 5, 5)$

L_1: $x = 7 + 2s$
$y = 2 + s$
$z = -6 - 3s$

L_2: $x = 3 + t$
$y = 9 + 5t$
$z = 13 + 5t$

Let x's equal each other for L1 & L2

$7 + 2s = 3 + t$
① $t = 4 + 2s$

Let y's equal each other for L1 & L2

$2 + s = 9 + 5t$
② $t = \frac{-7+s}{5}$

Let ① equal ②

$4 + 2s = \frac{-7+s}{5}$
$20 + 10s = -7 + s$
$9s = -27$
$s = -3$

sub $s = -3$ into L_1

$(x, y, z) = (7, 2, -6) + (-3)(2, 1, -3)$
$= (1, -1, 3)$

Point of intersection

3. Determine the equation of the line of intersection between the planes [5 MARKS]
$\pi_1: 2x + 3y - z - 3 = 0$ and $\pi_2: -x + y + z - 1 = 0$

① $2x + 3y - z = 3$
\+ ② $-x + y + z = 1$
$x + 4y = 4$

Let $y = s$

$x + 4s = 4$
$x = 4 - 4s$ & $y = s$

sub x & y into π_1

$2(4 - 4s) + 3(s) - z - 3 = 0$
$8 - 8s + 3s - z - 3 = 0$
$-z = -5 + 5s$
$z = 5 - 5s$

line of intersection

$\therefore$ $x = 4 - 4s$
$y = s$
$z = 5 - 5s$

4. Determine the point of intersection between the line $\ell: (x, y, z) = (2, -5, 3) + t(3, 1, 5)$ and the plane $\pi: 5x + y - 2z + 19 = 0$. [5 MARKS]

L: $x = 2 + 3t$
$y = -5 + t$
$z = 3 + 5t$

sub L into π

$5(2 + 3t) + (-5 + t) - 2(3 + 5t) + 19 = 0$
$10 + 15t - 5 + t - 6 - 10t + 19 = 0$
$6t = -18$
$t = -3$

sub $t = -3$ into L

$x = 2 + 3(-3)$
$= -7$

$y = -5 + (-3)$
$= -8$

$z = 3 + 5(-3)$
$= -12$

Point of intersection

$\therefore (-7, -8, -12)$

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APPLICATION

5. Determine the vector equation of the line that passes through the point $P(-1, 2, 5)$ and is perpendicular to the plane $3x - 4y + 2z - 7 = 0$. [3 MARKS]

$\vec{n} = [3, -4, 2]$

$(x, y, z) \cdot (3, -4, 2) = 0$

$3x - 4y + 2z = 0$ ✓

let $x = 1$ & $y = 1$

$3(1) - 4(1) + 2z = 0$

$z = \frac{1}{2}$

$\therefore \vec{m} = (1, 1, \frac{1}{2})$ ✓

$\therefore [x, y, z] = (-1, 2, 5) + s(1, 1, \frac{1}{2})$ ✓

6. In each case, state whether the system has no solutions, a unique solution (ie point of intersection), or an infinite number of solutions (ie line or plane of solutions). Explain. Do not solve the system! [9 MARKS]

(a) ① $2x - y + 3z = 5$
② $4x + y + z = -2$
③ $3x - y + z = 7$

$\vec{n_1} = (2, -1, 3)$
$\vec{n_2} = (4, 1, 1)$
$\vec{n_3} = (3, -1, 1)$ ✓ normals not parallel or coincident

① $2x - y + 3z = 5$
$3\times$② $12x + 3y + 3z = -6$
④ $-10x - 4y = 11$ ✓

② $4x + y + z = -2$
③ $3x - y + z = 7$
⑤ $x + 2y = -9$

④ & ⑤ are not parallel or coincident so there is a point of intersection ✓

(b) ① $2x - y + z = 1$
② $3x - 5y + 4z = 3$
③ $3x + 2y - z = -1$

$\vec{n_1} = (2, -1, 1)$
$\vec{n_2} = (3, -5, 4)$
$\vec{n_3} = (3, 2, -1)$ ✓ normals not parallel or coincident

① $2x - y + z = 1$
③ $3x + 2y - z = -1$
④ $5x + y = 0$ ✓

$4\times$① $\rightarrow 8x - 4y + 4z = 4$
② $\rightarrow 3x - 5y + 4z = 3$
⑤ $5x + y = 1$

④ & ⑤ are parallel so there is no solution ✓

(c) ① $3x - 9y + 3z = 9$
② $x - 3y + z = 3$
③ $2x - 6y + 2z = 12$

$\vec{n_1} = (3, -9, 3)$
$\vec{n_2} = (1, -3, 1)$ ✓
$\vec{n_3} = (2, -6, 2)$

$3\times$② = ① $\rightarrow$ $3\times$② $3x - 9y + 3z = 9$, ① $3x - 9y + 3z = 9$ } coincident ✓

$1.5\times$③ = ① $\rightarrow$ $1.5\times$③ $3x - 9y + 3z = 9$, ① $3x - 9y + 3z = 18$ } parallel

2 coincident planes and 1 parallel

no solution ✓

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7. Consider the following system of equations:

① $2x + y + 6z = p$

② $x + my + 3z = q$

Determine the values of m, p and q such that the two planes.

(a) are coincident. [3 MARKS]

① $(2,1,6)$

② $(1,m,3)$

$2 \times ① = ②$

$2(1,m,3) = (2,1,6)$ ✓

$(2,2m,6) = (2,1,6)$

$2m = 1$

$m = \frac{1}{2}$ ✓

$\boxed{p = 2 \quad q = 1}$ ✓

(b) intersect at right angles. [3 MARKS]

$(2,1,6) \cdot (1,m,3) = 0$ ✓

$2 + m + 18 = 0$

$m = -20$ ✓

$\boxed{p = 0 \quad q = 0}$ ✓

8. Determine the solution to the following system of equations: [10 MARKS]

① $x - y + z = 1$

② $x + y + 2z = 2$

③ $x - 5y - z = 1$

$\vec{n_1} = (1,-1,1)$

$\vec{n_2} = (1,1,2)$

$\vec{n_3} = (1,-5,-1)$ ✓

not parallel or coincident ✓

$-1 \times ① \rightarrow -x + y - z = -1$

$\oplus \; x + y + 2z = 2$

④ $2y + z = 1$ ✓

$-1 \times ① \rightarrow -x + y - z = -1$

③ $\rightarrow x - 5y - z = 1$

$\oplus$

⑤ $-4y - 2z = 0$ ✓

$2 \times ④ \rightarrow 4y + 2z = 2$

⑤ $\rightarrow -4y - 2z = 0$

$0y + 0z = 2$ ✓

no solution ✓

$\vec{m_1} = \vec{n_1} \times \vec{n_2}$

$= [(-1)(2) - (1)(1), (1)(1) - (1)(2), (1)(1) - (-1)(1)]$

$= (-3,-1,2)$ ✓

$\vec{m_2} = \vec{n_1} \times \vec{n_3}$

$= [(-1)(-1) - (1)(-5), (1)(1) - (1)(-1), (1)(-5) - (-1)(1)]$

$= (6,2,-4)$

$= -2(-3,-1,2)$ ✓

$\vec{m_3} = \vec{n_2} \times \vec{n_3}$

$= [(1)(-1) - (2)(-5), (2)(1) - (1)(-1), (1)(-5) - (1)(1)]$

$= (9,3,-6)$

$= -3(-3,-1,2)$ ✓

$\therefore$ the systems of equations are inconsistent and form a triangular prism. ✓

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